A game theoretic approach to strikes and negotiations

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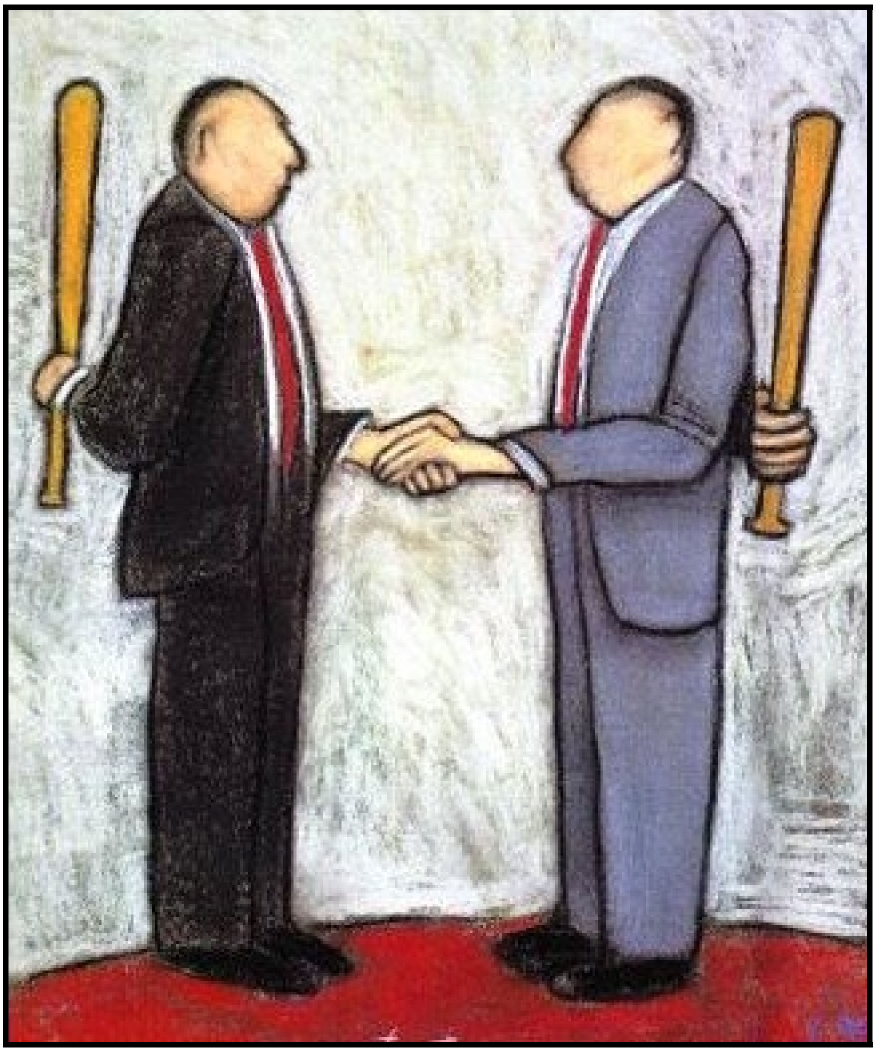

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Abstract

This thesis discusses strikes and negotiations from a perspective of bargaining theory using perfect information. The more general notion of surplus destruction is used throughout the thesis and describes the situation—arising from, for instance, strikes—in which some or all of the surplus is destroyed.

Part I summarizes and discusses the existing literature. This includes a brief introduction to the game theoretic framework and a presentation of the Rubinstein model. We then proceed to essentially two different models: the models by Fernandez & Glazer and by Busch, Shi & Wen, respectively, serve as the cornerstones of the analysis.

Part II presents our own results. These include a more general model that includes Fernandez & Glazer's as well as Busch, Shi & Wen's models as extremes and a continuous transition between the two. Further, we add surplus regeneration to the model where the firm has the possibility of regenerating some of the surplus destroyed by the union. We show that equilibria with surplus regeneration exist in both Fernandez & Glazer and Busch, Shi & Wen.

Resumé på dansk

Denne afhandling diskuterer strejker og forhandlinger med forhandlingsteori og perfekt information som udgangspunkt. Igennem afhandlingen bruges det mere generelle begreb profit ødelæggelse til at beskrive den situation, der for eksempel opstår ved strejker, hvor en del af eller hele profitten ødelægges.

Del I opsummerer og diskuterer den eksisterende litteratur på området. Den inkluderer en kort indledning til den spilteoretiske tankegang samt en præsentation of Rubinsteins model . Derefter behandles to essentielt forskellige modeller, hvor henholdsvis Fernandez og Glazer og Busch, Shi og Wen danner grundlag for analysen.

Del II præsenterer vores egne resultater. Dette inkluderer en mere generel model, der indeholder Fernandez og Glazers såvel som Busch, Shi og Wens model som ekstreme tilfælde samt giver mulighed for en kontinuert overgang mellem disse. Derudover indføres profit regenerering i modellen, hvilket giver firmaet mulighed for at regenerere noget af den profit som fagforeningen har ødelagt. Vi viser, at der eksisterer ligevægte med profit regenerering i såvel Fernandez og Glazer såvel som Busch, Shi og Win.

Preface

From the beginning we clearly knew that we wanted to investigate current topic. Further, we wanted the underlying theory to be elegant and powerful. We quickly agreed to use game theory with its concise formulation and myriad applications. With both authors having strong ties to Germany, it was natural to consider the economic issues Germany currently faces and, among these, the German National Railway strike seemed very interesting.

Writing an extensive scientific paper is challenging and has spawned many frustrations. In the beginning we spent long hours doing initial research and becoming familiar with the existing models. The work on our model has taken a trial-and-error approach in which we investigated many ideas only to find out that they were very fruitless. Nevertheless, we feel that this process has matured us immensely and helped us gain at least some understanding of the research methods used in mathematical economics.

Some people have contributed to help making the writing process a pleasant one. First, we thank our patient adviser Eliot Maenner, who always answered questions, arranged meetings and provided fruitful discussions. His suggestions helped us regain our motivation during the long hours.

Further, David Breuer's editing has improved the quality. More importantly, however, it brings pleasure to the reader and helps to convey the intended message. We thank him sincerely for his help and for his humor and mathematical precision in language.

Jerôme Baltzersen and Dominik Dienst
Copenhagen, July 2008

CHAPTER 1

Introduction

> "*The difficulty is to understand why rational parties should resort to a wasteful mechanism as a way of distributing the gains from trade. Why could not both parties be made better off by moving to the final distribution of surplus immediately... and sharing the benefits from increased production?*"
> (Oliver Hart [1], 1989)

This statement by Oliver Hart tackles a problem that many economists have difficulty explaining. Following the principle that people think rationally, one might suggest that a rational person would never use a tool that leaves another person with a more or less temporary handicap. Considering people that go on strike as being rational, the question arises why rational parties would waste some of their share to achieve their individual aims? Nevertheless, a strike is known to be a resource-destroying action that is widespread in all kinds of economies. It is thus essential for economists to understand the justification of strikes in an economic perspective. Recent works by economists have tried to explain the phenomena. Our thesis analyzes strikes in depth using economic game theory. Game theory enables us to set up the strike models in an economic game analyzing the utility and outcome of different strategies used by the involved parties, who could have opposing interests.

Strikes exist in many forms. The best-known strike method is the general strike: withdrawing all labor power to hurt the firm. This definitely puts the firm into a position where it cannot produce, which usually results in lost revenue and thereby profit. However, other interesting types of strikes also reduce profit. One is called the Japanese strike, in which workers are excessively productive and produce more output than needed. The additional output then must be stored (or even thrown away in food production), which reduces profit. This method is especially effective today since many supply chains are just-in-time systems. Another possibility is a sickout. This method is used by groups of workers that are not allowed—either by law or their contract—to strike. Examples include employees essential to society such as police officers or fire fighters. In a sickout, the workers call in sick and stay home. Yet another method is the work-to-rule strike, where workers comply very strict to any or all requirements in their contract. For instance they might avoid working overtime, helping co-workers, putting in extra effort when needed etc. It might also include working in strict accordance to health and safety rules, thereby limiting productivity immensely. This can slow down production and thus put the firm under pressure. However, not every strike has the desired outcome. An example in California shows that fruit pickers striking caused was a serious fruit shortage. Prices started to rise because the demand could not be met. This in return favored the employers because earnings increased. Thus, the strike was fruitless and the fruit pickers soon went back to work.

In Europe, unions are still a great economic and political power. They are an important actor in negotiating wage contracts and are not afraid to use their unity of workers to call for strikes. However, strikes are commonly used in wage negotiations not only in Europe but also in the United States, where unions are not as common as in Europe. Recent examples are the strike of the General Motors working staff in 2007 or the strike of the Writers' Guild in 2007–2008.

Striking as a means to increase bargaining power or to at least ensure existing contracts is no new phenomena. The first strike recorded goes back to the year 1155 A.D. and is known as the Strike of Deir el-Medineh in Egypt. Workers were building the grave of Ramses III and his family. Their wage was paid in food, and in October 1155 the workers went on strike since they had not been paid their ration for 18 consecutive days. With the slogan "We are hungry" they tried to get attention and to stress that they would not continue to work if they were not paid according to the contract. The strike lasted for seven days and, as a result, the workers got paid their ration for the entire month of October. This shows the importance of strikes and their success in bargaining situations and that they have been used for millennia.

Our thesis presents two cases to provide some empirical findings related to the theoretic results presented. Our cases are based on two strike situations. One took place in Germany by the German Engine Drivers' (GDL) from July 2007 until March 2008. The GDL sought to increase the engine drivers' wage paid by the German National Railways (DB) by using broad national strikes as a tool to put DB under pressure. During a strike, DB loses revenues. However, in this example the DB and the GDL were not the only ones affected by the strike since the DB provides public passenger transport and serves as a transport agent for companies. Thus, the strike affects the economy as a whole, reducing the gross domestic product. Therefore legislation of many countries is such that politicians can interfere in negotiations forcing the strike to come to a stop. This lowers the negotiation power of the union. Denmark is notorious for not having the politicians interfere and let the labor market self-adjust. This attitude by the Danish government is also seen in the two cases we analyze.

Not every strike affects the economy as a whole. Nevertheless, every strike affects some third party not directly involved in the strike itself. Our second case, the strike by the Danish Nurses' Organization in Denmark in 2008, is another example of this. Here again, this group of workers is critical since their striking affects other people who need health care. The strike began on April 16 and an agreement was reached on June 25. During this times everyone with a non-life-threatening illness was put on a waiting list. It is thus a matter of high priority for economists to understand the structures of strikes and their justification.

The groundwork of bargaining that can be used in analyzing the theory of strikes in a game theoretic way was established by Nash [2][3] in 1950 and later developed further by Rubinstein [4] among others.[1] Nash has not only lent his name to the infamous Nash equilibrium, which states the best strategies as a response to another player's strategies, he also introduced the formal axiomatic bargaining game that is still used today. Rubinstein gave a formal definition of the bargaining game with perfect information in 1982. His definition of the game is the basis for many of subsequent papers focusing on the problem of bargaining or its relatives. Rubinstein's bargaining game is also fundamental for the studies of strike theory. We later review his original paper to provide the reader with basic understanding of the bargaining used in the strike models discussed in the main body of this thesis.

Besides the theory of bargaining models, other theories that be used in discussing models of strikes. A strike model could be embedded in a pure repeated game. See, for example Maria Paz Espinosa & Changyong Rhee (1989). In a repeated game a union and a firm play the same type of game again and again, proposing wage contracts and then choosing the number of workers to be employed, respectively. However, surplus-destroying actions such as strikes require some degree of asymmetric information. Models including asymmetric information distort the possibility of strikes as a surplus-destroying activity. We discuss models of asymmetric information more extensively below. Another model is the war of attrition. See, for example, *Evolution and theory of Games* by John Maynard Smith (1982). In the war of attrition, two players are fighting about resources. In its origin, a war of attrition is a version of an all-pay sealed-bid second-price auction.[2] In this game, time is continuous and it is easily shown that, in any pure Nash equilibrium, all players but one will forfeit immediately: that is, there is no delay. However, if mixed strategies are allowed, it is possible to support (real time) delay in a Nash equilibrium. Despite this rather simply way of constructing a delay, the war of attrition game does not capture the negotiation aspect at all.

The literature on strikes grew explosively during the 1980s and the 1990s. The first models were based on asymmetric information. This assumption is based on the idea that, during wage bargaining, some information might be private to one player. One example is to think of the union not knowing the profit of a firm. Bargaining models with asymmetric information have been developed by Anat Admati & Motty Perry (1987), Kalyan Chatterjee & Larry Samuelson (1987), Lawrence M. Ausubel & Raymond J. Deneckere (1989), Peter C. Cramton (1984), Drew Fudenberg et al. (1985), Sanford, Grossman & Perry (1986), Oliver Hart (1989), Ariel Rubinstein (1989), and Joel Sobel & Ichiro Takahashi (1983). These models all say that strikes serve as a signalling device. A high-profit firm might be more willing to delay agreement, whereas a low-profit firm might not have the privilege to endure much delay. This is because a high profit firm might find it more profitable in the long run to have employees work for less compared to the loss of profit due to the strike. Thus, the willingness to delay agreement signals the firm's bargaining strength.

In our thesis, we do not see strikes as a signal, but as a real threat to destroy surplus. Our analysis is mainly based on the articles of Raquel Fernandez & Jacob Glazer [5] and Lutz-Alexander Busch, Shouyong Shi & Quan Wen [6]. They use models of complete and perfect information in which the possibility to strike is part of the players' strategy and serves as a threat. Our analysis is supplemented with

[1] Actually, Frederik Zeuthen, a Dane, formulated a version of his bargaining model as early as in 1930.

[2] All-pay: any player that enters the auction and does not win has strictly negative payoff. Sealed-bid second-price (also known as Vickrey auction): the winner pays the second highest bid.

the works of Christopher Avery & Peter Zemsky [7], Paolo Manzini [8] and Wilko Bolt [9]. We show how including strikes even allows for inefficient equilibria.

We decided to analyze models involving perfect information for two reasons. Firstly, models involving asymmetric information tend to increase rapidly in complexity and thereby make the results obscure. Secondly, we strongly believe that, even though asymmetric models may more accurately describe the real world a perfect information framework can lead to much understanding and many interesting results. Further, many companies listed on the stock exchange or public institutions are required to provide detailed information about their structure and present economic situation. We also believe that, although some findings have already been made in this field, it is by no means exhausted.

Our initial research revealed essentially two different approaches within the framework of perfect information. We found it helpful to classify these as temporary and permanent surplus destruction, respectively. This is discussed in detail in Chapter 4 and 5, where a strict definition is also found. Within these two approaches we found that Fernandez & Glazer and Busch, Shi & Wen, respectively, had the most general models offering the deepest analysis and in many cases even justifiable assumptions of the simpler models. Using these as cornerstones in the analysis therefore seemed natural. We still refer to the other papers, and their discussion is often fruitful in the analysis.

This thesis is divided into two parts. Part I contains a brief Chapter 2 presenting the game theoretic framework needed for future discussion. Chapter 3 presents the Rubinstein model, which has a unique and efficient subgame perfect equilibrium. Although it is a very simplistic model, it is imperative to any further analysis as it often serves as a benchmark equilibrium. Not only is it contained as a special case in all the other models, but it is also used as a trigger strategy to prove the existence of new equilibria. We then describe the Fernandez & Glazer model as well as the Busch, Shi & Wen model. They both incorporate strikes, thus increasing the union's bargaining strength and hence its payoff share. Introducing strikes into the model enables inefficient (such as delayed) equilibria to be generated as motivated empirically.

Part II presents our model. This essentially offers two classes of results. Firstly, it is a generalization that includes both Fernandez & Glazer and Busch, Shi & Wen as extreme special cases. Due to our more abstract formulation, it becomes possible to analyze a "mixture" of the two models. Secondly, it introduces the concept of surplus regeneration. The firm can regeneration surplus after the union has destroyed surplus. This is modeled as an opting-out option, and we prove that equilibria with non-zero surplus regeneration exist. To end Part II, we conclude on our findings and offer perspectives on questions of future interest that lie beyond the scope of this thesis.

Part I

Review of existing literature

CHAPTER 2

Basic concepts of game theory

2.1 Introduction

This chapter introduces the basic concepts of game theory used in this thesis. Its purpose is primarily to introduce the notation and a few theorems to which we refer later. This chapter is not meant to teach the reader game theory. Our primary references are Mas-Colell et al. [10] and Osborne & Rubinstein [11]. They offer different perspectives, and their notation varies slightly. Nevertheless, we tried to produce a condensed version of what we need, which can serve as a cornerstone for what lays ahead.

2.2 Strategic game

First, we need to define a strategic game and concepts related to it.

Definition 2.1 (strategic game). *A strategic game consists of a set of players N, the set of possible actions for player $i \in N$ denoted by A_i and a rational preference relation $\succeq_i$ on $A = \times_{i \in N} A_i$ for the ith player. If A_i is finite for each i, we say that the game is finite. A compact notion for the game is $\langle N, (A_i)_{i=1,\dots,N}, (\succeq_i)_{i=1,\dots,N} \rangle$ or simply $\langle N, (A_i), (\succeq_i) \rangle$. If $\succeq_i$ is continuous, it has a utility function representation given by $u_i : A \to \mathbb{R}$; this function is often referred to as the payoff function. If it exists for all i, we might denote a game by*

$$\langle N, (A_i), (u_i) \rangle, \tag{2.1}$$

which is equivalent to our first statement. An element $a \in A$ is called a strategy profile or outcome.

In the theory that follows and in applications, we assume that we have a payoff function representation.

Let the best-response function for player i, $B_i : A_{-i} \to A_i$, be a set-valued function[1] given by

$$B_i(a_{-i}) = \{a_i \in A_i : (a_{-i}, a_i) \succeq_i (a_{-i}, a_i^{'}) \quad \forall i \in A_I\}.$$

Further, let the generalized best-response function, $B : A \to A$, be given by $B(a) = \times_{i \in N} B_i(a_{-i})$. The Nash equilibrium formalizes the idea of an interdependent optimal choice among the N players.

Definition 2.2 (Nash equilibrium). *A Nash equilibrium is a (strategy) profile $a^* \in A$ for which*

$$a_i^* \in B_i(a_{-i}^*) \tag{2.2}$$

for all $i \in N$ or, equivalently, there exists a^* such that

$$a^* \in B(a^*). \tag{2.3}$$

We now proceed to show the existence of a Nash equilibrium. The equivalence of (2.2) and (2.3) follows from the definition of B. This observation is used to prove the following theorem.

Theorem 2.3. *The strategic game $G = (N, (A_i), (\succeq_i))$ has a Nash equilibrium if, for all $i \in N$, A_i is a nonempty compact and convex subset[2] of $\mathbb{R}^n$ and the $\succeq_i$ fulfills the following:*

1. *continuous; and*
2. *quasi-concave on A_i; that is, for every $a^* \in A$, the set $\{a_i \in A_i : (a_{-i}^*, a_i) \succeq_i a^*\}$ is convex.*

Proof. We want to show the existence of a fixed point for B by applying Kakutani's fixed-point theorem.[3] Our assumptions about $\succeq_i$ guarantee that $B(a)$ is convex. It is nonempty, as every $B_i(a_{-i})$ is nonempty. This follows because $\succeq_i$ is continuous and A_i is compact; hence the utility maximization problem (UMP) has a solution. Continuity of $\succeq_i$ implies that B's graph is closed. □

Note that A_i being nonempty means that each player has at least one strategy to play. Further, in a strategic game G as considered in Theorem 2.3, utility function representation will always be available. The theorem proves that a Nash equilibrium for a strategic game exists if each player has at least one strategy to play—that is, A_i is nonempty (and compact and convex)—and that each player's preference relation $\succeq_i$ is continuous and quasi-concave.

2.3 Extensive games and subgame perfect equilibrium

Whereas the strategic games featured a once-and-for-all independent choice of action by each player at the beginning of the game, an extensive game describes the sequential structure often faced in games. This naturally suits our purpose of describing a negotiation process. Extensive games can have perfect or imperfect information.

[1] Consult Appendix A for a brief discussion of set-valued functions.
[2] A generalization would be to consider nonempty compact and convex subsets of an Euclidean space. Any Euclidean space will always be isomorphic to $\mathbb{R}^n$ for some n. This thesis is therefore restricted to $\mathbb{R}^n$.
[3] See appendix A Theorem A.1

In a game with perfect information, each player is perfectly informed of all previous actions—his or her and others—when making a decision. One example of such a game could be chess. In an extensive game with imperfect information, the player making a decision may not know some or all of the preceding history; he or she may even forget actions he or she took earlier in the game.[4]

The main goal of this section is to define subgame perfect Nash equilibrium (SPE)—a much stronger concept of equilibrium. This concept is vitally important in building the theory to come.

Definition 2.4 (extensive game)**.** *An extensive game, Γ, is given by $\Gamma = \langle N, H, P, (\succeq_i)\rangle$, where N is the set of players and H is the set of histories. Each $(a_j)_{j \leq L} = h \in H$ for some $L \in \mathbb{N} \cup \{\infty\}$ provides information on what actions the involved players took earlier in the game, and each term of any history is an action taken by a player during the game: that is, $a_j \in A_i$ for some $i \in N$. Further, if $(a_j)_{j \leq L} \in H$, then $(a_j)_{j \leq L'} \in H$ for any $L' \leq L$. A history is terminal if there are no further actions to be taken. The set of terminal histories is denoted by Z. All non-terminal nodes are also called decision nodes. Each of the nodes is assigned to an information set, and $\mathfrak{H}$ denotes the collection of information sets and $\mathfrak{H}_i$ denotes the information set of player i.[5] The function $P : H \to N$ assigns to each history the player who will move next, and $A(h) := \{a : (h, a) \in H\}$ is the set of actions available to player $P(h)$. Finally, $\succeq_i$ is a preference relation on Z.*

In a game with perfect information, a player is certain of which preceding history h has been played. That is, every information set is a singleton. A game of imperfect information has at least two histories $h = (a_1, \ldots, a_L, a_{L+1})$ and $h' = (a_1, \ldots, a_L, a'_{L+1})$ for which $h \neq h'$ and $P(h) = P(h')$ such that player $P(h)$ does not know at which of the two or more decision nodes he or she is: that is, the information set has two or more members.

Lastly, for $i \in N$, a strategy $s_i : \mathfrak{H}_i \to A(h)$ selects a particular action available to player i after the history h from $A(h)$. A strategy profile is $s = (s_i)_{i \in N}$. Let $O(s)$ be the outcome: that is, the payoff of the particular terminal node induced by the strategy profile s.

The sequential nature of the extensive game is described using the set of histories H. Further, a strategy s_i assigns an action to each and every information set *even if previous actions within the strategy rule out the information set.* The definition of SPE makes the reason for this apparent. Due to the sequential nature of the game, parts of the game can be regarded as independent games in their own right; these are called subgames.

Definition 2.5 (subgame)**.** *Given $(a_j)_{j \leq L} = h \in H$ such that a_L is the only member of the information set, a subgame of Γ following $h \in H$, $\Gamma(h)$ is a subset of the game that inherits the structure of Γ. With $H|_h := \{(h, h') \in H\}$, $P|_h(h') := P(h, h')$ and $\succeq_i |_h$ defined by $h' \succeq_i |_h h''$ if and only if $(h, h') \succeq_i (h, h'')$, we arrive at $\Gamma(h) = \langle N, H|_h, P|_h, \succeq_i |_h\rangle$.*

The requirement that a_L be the only member of the information set guarantees that any subgame is a game in its own right, and we further note that Γ is a subgame of itself, namely $\Gamma(\emptyset)$. We are now ready to define a stronger concept of equilibrium: the subgame perfect equilibrium.

[4]A game in which all players never forget their previous actions is called *perfectly recallable.*

[5]Technically speaking, the information sets are a partition of the decision nodes. If two nodes are members of the same information set, the player choosing the next action will not be able to distinguish between them.

Definition 2.6 (Nash equilibrium and subgame perfect Nash equilibrium of extensive game). *A Nash equilibrium of an extensive game is a strategy profile s^* such that*

$$O(s_i^*, s_{-i}^*) \succeq_i O(s_i, s_{-i}^*) \quad \textit{for all } i \in N \textit{ and all } s_i.$$

A strategy profile s for an extensive game Γ is an SPE if it induces a Nash equilibrium in every subgame $\Gamma(h)$.

Note first that any SPE is also a Nash equilibrium. Consider a strategy $s = (s_1, s_2)$, which is an SPE. If s_2' differs from s_2 only in nodes that are never reached when player 1 plays according to s_1, then $s' = (s_1, s_2')$ will also be a Nash equilibrium but not necessarily an SPE. Considering Definition 2.6, it seems rather cumbersome to check whether a given strategy is an SPE or not. Fortunately, the following lemma eases this task.

Lemma 2.7 (The one deviation property). *For any given finite horizon extensive game $\Gamma = \langle N, H, P, (\succeq_i)\rangle$, a given strategy s^* is an SPE in Γ if and only, if for any history $h \in H$, player i cannot improve his or her outcome by changing only his or her action immediately following h. That is,*

$$O(s_i^*|_h, s_{-i}^*|_h) \succeq_i O(s_i|_h, s_{-i}^*|_h) \quad \textit{for all} \quad s_i|_h. \tag{2.4}$$

Proof. Consult p. 98 in Osborne and Rubinstein [11]. □

The games on which we focus in this thesis have an infinite horizon, and thus we need the following.

Corollary 2.8 (the one deviation property for infinite-horizon extensive games). *The one deviation property formally described in (2.4) continues to hold for infinite-horizon games if the outcome stabilizes sufficiently rapidly: that is, there exist an $N \in \mathbb{N}$ such that, for all $\varepsilon > 0$,*

$$\left|O(s_i|_{h'}, s_{-i}^*|_{h'}) - O(s_i|_h, s_{-i}^*|_h)\right| < \varepsilon \quad \textit{for all } s_i|_h \textit{ and } s_i|_{h'},$$

where $l(h') = l(h) + 1 \geq N$, where l denotes the length of a given history and s^* is the equilibrium strategy profile.[6]

Proof. The proof that the one deviation property also holds for decreasing infinite-horizon games is based on the fact that preferences are continuous and can thus be represented by a continuous utility function u. Hence, for any given ε, one can truncate a game after a certain number of periods N, and the difference in outcome between the possible one-step deviations from s^* will be smaller than ε. □

Note that, in the bargaining game described in Chapter 3, the outcomes are represented by a infinite convergent geometric series, which decreases sufficiently rapidly in the sense of Corollary 2.8.

[6] In which we assume that the outcome function maps to the real numbers.

CHAPTER 3

Rubinstein bargaining model

3.1 Bargaining and game theory

Frederik Zeuthen, who lived in Denmark, formulated a version of his bargaining model in 1930. In 1950, Nash [2] introduced his axiomatic approach to bargaining in a game theoretic framework, which is acknowledged as the basic foundation in bargaining theory today. Bargaining imposes a game on a number of players in which the players' well-being can only be affected if all players agree to the proposed action. In his famous article, Nash considered the special case of a two-player game. Trading between two countries or wage bargaining between a firm and a union are examples of bargaining games. This thesis focuses on the latter.

In the following section, we consider the two-player bargaining game as Ariel Rubinstein presented it in his 1982 article [4], whereas our presentation follows along the lines of Osborne & Rubinsteins textbook presentations [11][12]. The main result of this chapter is Theorem 3.2, which proves the existence of a unique SPE in the bargaining game. Our analysis has its roots in the theory of extensive games with perfect information presented in section 2.3. In contrast to the axiomatic approach by Nash, Rubinstein uses a strategic approach in his model. This thesis only focuses on the strategic approach, but for broader insight into the axiomatic approach of bargaining models, consult Roth [13] and Osborne & Rubinstein [12].

3.2 The Rubinstein bargaining model

We begin with an example, which the reader should keep in mind while reading the more abstract material that follows. In extensive games with perfect information, players take their actions one at a time as a response to previous players' actions.

Example (split-the-euro). Consider a situation in which two bargainers want to split one euro. In the first period, player 1 makes a proposal and player 2 decides whether to accept or reject the offer. If player 2 accepts, the game ends with the proposed division of the euro. If player 2 rejects, the game continues to period 2, where player 2 suggests a split and player 1 needs to accept or reject the split. Again, if player 1 accepts the split, the game ends, whereas if player 1 rejects it the game continues to period 3, in which player 1 again makes the proposal and player 2 has to accept or reject it. The number of periods may be finite or infinite.[1]

We want to formalize this game: define a compact and connected subset X of $\mathbb{R}^2$. Thus, a proposal $x = (x_1, x_2)$ is a member of X. The set of players is $\{1, 2\}$ and the responding player can accept the offer (Y), or reject it (N). Without loss of generality, let us assume that player 1 makes offers in even periods and responds to offers in odd periods. Analogously, player 2 proposes a split in odd periods and accepts or rejects an offer in even periods. A period is measured in a unit of time and is denoted by $t \in T$, where $T \subseteq \mathbb{N}_0$. To define the set of histories H, let $x_t \in X$ be the proposed partition at time t. There are four possible types of sequences:

1. sequences that have an offer as their last object: $(x_0, N, x_1, N, ..., x_t)$;
2. sequences that have rejection as their last object: $(x_0, N, x_1, N, ..., x_t, N)$;
3. sequences that have acceptance as their last object: $(x_0, N, x_1, N, ..., x_t, Y)$; and
4. sequences that go to infinity $(x_0, N, x_1, N, ...)$.

H is then the set of all such sequences. The sequences of type 1 and 2 are non-terminal histories. After a type 1 history, one player has to accept (Y) or reject (N) the proposal, whereas after a type 2 history the next player has to make his proposal. Sequences of type 3 and 4 are terminal histories, since the game ends with the accepted agreement x_t or disagreement, respectively. Further, for every $h \in H$ we define without loss of generality the player function $P : h \to N$, where $P(h) = 1$ whenever h is a history of type 1 or 2 and t is even and, similarly, $P(h) = 2$ whenever h is a history of type 1 or 2 and t is odd.

Consider the preference relation $\succeq_i$, $i = 1, 2$, over the possible outcomes of the game. We introduce a set of discontinuity factors $\delta_i \in (0, 1)$, for $i = 1, 2$, which implies that the players prefer to receive the same outcome sooner than later: that is, time is valuable. Let us specify the outcomes at the different terminal histories (type 3 and 4 histories): for each $(x, t) \in X \times T$, the outcome of type 3 histories for player i is x_i and the utility function is $u_i(x, t) := \delta_i^t x_i$. The preference relation for player i is given by

$$(x, t) \succeq_i (x', t') \quad \text{if and only if} \quad \delta_i^t x_i \geq \delta_i^{t'} x_i'. \tag{3.1}$$

Further, we assume path independence: that is, players are only interested in the size of an agreement and the time at which it was made, which means that all the previously rejected proposals do not influence the preference relation $\succeq_i$. In particular, any disagreement will feature the same outcome, denoted by D and $u_1(D, t) = u_2(D, t) = 0$, for all $t \in T$: that is, both players receive nothing when disagreeing. $\succeq_i$ is thereby player i's preference relation over the set $(X \times T) \cup D$.

The following four conditions for player i's preference relation $\succeq_i$ given by (3.1) are satisfied:

(A1) [Disagreement is the worst outcome] $(x, t) \succeq_i D$ for all $(x, t) \in X \times T$.

[1] Usually, the literature calls this example *split-the-pie*.

(A2) [Time is valuable] $(x, t) \succeq_i (x, t+1)$ for all $x \in X$ and $t \in T$ where the preference is strict if $(x, 0) \succeq_i D$.

(A3) [Stationary preferences] $(x, t+1) \succeq_i (y, t)$ if and only if $(x, 1) \succeq_i (y, 0)$ and $(x, t) \succeq_i (y, t)$ if and only if $(x, 0) \succeq_i (y, 0)$: that is, preferences between (x, t) and (y, s) depend only on the outcomes x and y and the difference in time, $t - s$.

(A4) $\succeq_i$ is continuous.

Using the notation just introduced a general bargaining game is defined as follows.

Definition 3.1 (bargaining game)**.** *The bargaining game of alternating offers is understood to be the extensive game with perfect information denoted by* $(N, H, P, \succeq_i)$ *or, equivalently,* (N, H, P, u_i).

Theorem 1 and 2 in Fishburn & Rubinstein [14] proves that any preferences fulfilling (A1) through (A4) have a utility function representation of the form $u_i(x, t) = \delta_i^t x_i$.

To complete the formalization of our split-the-euro game, note that $X := \{x \in [0, 1]^2 : x_1 + x_2 = 1\}$. We spend substantial time analyzing this game during the remainder of this thesis. First we consider what Nash equilibria such a game supports. For any $x^* \in X$, a pair of strategies $s = (s_1, s_2)$ exist such that the outcome of s is $(x^*, 0)$ and s is a Nash equilibrium. The pair is given by player 1 always suggesting x^* and accepting an offer x if and only if $x_1 \geq x_1^*$. Similarly, player 2 proposes x^* and accepts an offer x if and only if $x_2 \geq x_2^*$.[2]

3.3 Subgame perfect equilibrium

We have just seen that the notion of Nash equilibrium is rather weak, as any outcome is supported. Further, it can easily be shown that the above-mentioned strategies do not in general constitute an SPE, since the strategies must induce a Nash equilibrium in every subgame. Consider the situation in which player 2 rejects player 1's initial offer x and thereafter proposes an offer x' with $x_1 > x_1' > \delta_1 x_1$. Player 1 should accept this offer, since by rejecting it to receive x_1 in the next period he or she will only receive an outcome of $\delta_1 x_1$. Thus, the above strategies are not an SPE since they do not induce a Nash equilibrium in every subgame.

We now shift our attention to finding an SPE of the split-the-euro game. These concepts are easily generalized to the bargaining game, but doing so is unnecessary for the purpose of this thesis. First, we define the *Pareto frontier* of the set X to be the agreements x for which there is no agreement y, with $(y, 0) \succ_i (x, 0)$ for player i and $(y, 0) \succeq_j (x, 0)$ for player $j \neq i$. An agreement located on the Pareto frontier is referred to as an *efficient* or *optimal* agreement. Note that all agreements in the split-the-euro game are efficient.

We are now ready to discuss the main result of this chapter, Theorem 3.2, but first we introduce some further properties (B1) through (B4) on the players' preference relations, which will later serve as the assumption for Theorem 3.2. It can easily be checked that (B1) through (B4) are satisfied for the preference relations established in the split-the-euro game given by (3.1).

[2] Actually, for any x^* in which agreement is reached in round $t \in \mathbb{N}_0$ a strategy profile exists such that x^* is a Nash equilibrium outcome.

(B1) No two agreements $x \neq y$ satisfy $(x, 0) \sim_i (y, 0)$ for both $i = 1, 2$.

(B2) $(z^i, 0) \sim_j (z^i, 1) \sim_j D$ for $i = 1, 2, j \neq i$, where z^i is the best agreement for player i: that is, $z^i = \sup\{x \in X : (x, 0) \succeq_i (x', 0) \forall x' \in X\}$.

(B3) [Monotonicity of the Pareto frontier]: If an agreement $x \in X$ is efficient, then no agreement $y \neq x$ exists such that $(y, 0) \succeq_i (x, 0)$ for both players i.

(B4) A unique efficient pair of agreements (x^*, y^*) exists such that

$$(x^*, 1) \sim_1 (y^*, 0) \quad \text{and} \quad (y^*, 1) \sim_2 (x^*, 0). \tag{3.2}$$

Note that, in the split-the-euro game, (3.2) corresponds to solving $\delta_1 x_1^* = y_1$ and $\delta_2 y_2 = x_2$ where $x_1 + x_2 = y_1 + y_2 = 1$, which yields the unique solution

$$x_1^* = \frac{1 - \delta_2}{1 - \delta_1 \delta_2} \quad \text{and} \quad y_2^* = \frac{1 - \delta_1}{1 - \delta_1 \delta_2}. \tag{3.3}$$

Note that x_1^* is increasing in δ_2 while y_2^* is decreasing. That is, as player 2 gets more and more impatient, his or her share decreases while player 1's share, ceteris paribus, increases. This result, formalized in the following theorem, is fundamentally important in the models we present later, and we often refer to it as the Rubinstein share. The celebrated result of the Rubinstein bargaining model is the unique SPE found above and formalized below.

Theorem 3.2 (subgame perfect equilibrium in a bargaining game of alternating offers). *A bargaining game as defined in Definition 3.1 that satisfies (B1) through (B4) has an SPE. Let (x^*, y^*) be the unique pair of efficient agreements that satisfy (3.2). Assume that player 1 makes the first offer. In every SPE, player 1 always proposes x^* and only accepts any proposal y such that $(y, 0) \succeq_1 (y^*, 0)$; player 2 always proposes y^* and only accepts any proposal x such that $(x, 0) \succeq_2 (x^*, 0)$. Thus the outcome is (x_1^*, y_2^*) as given in (3.3).*

Proof. Consult p. 122 in Osborne & Rubinstein [11]. □

If player 2 proposes first, the outcome would be $\left(\delta_1 \frac{1-\delta_2}{1-\delta_1\delta_2}, \frac{1-\delta_1}{1-\delta_1\delta_2}\right)$ with analogous strategies.

3.4 Critique of the Rubinstein model and purpose of an expansion

The Rubinstein model is celebrated for its unique SPE and thus has an appealing predictive aspect. As we have seen, this equilibrium is always agreed to in the first period of bargaining, which seems to contradict at least some empirical observations. Some negotiation processes are resolved immediately or rapidly and hence qualify for a description using the Rubinstein model. However, in many situations—especially those that are our primary interest—negotiations may break down for an extended period of time and a strike or similar destructive behavior is seen. The Rubinstein model is not suited to handle these situations.

A more advanced model should offer some sort of explanation for inefficiency, as these features are observed in real life. Inefficient equilibria can arise in three ways:

i) due to discounting (delayed agreement), ii) some surplus is destroyed[3] or iii) a combination of the two. The Rubinstein model only allows for i), and Theorem 3.2 indicates that this is not sufficient to support inefficiency.

How do we obtain the desired features in a model? The Rubinstein model is clearly not suitable for describing all negotiation purposes. Disagreement is simply taken to be a polite rejection of the other's offer but with no acts of consequence as such. It is not possible to strike and, more generally, surplus destruction is not possible. This behavior is destructive in nature, and a priori it is not clear why we should include it in a model, but empirical evidence suggests that this does exist as a consequence of heated discussions. By giving the involved parties new actions such as those mentioned above, we take a step closer towards a model that may potentially offer an economic description of negotiation processes involving destructive actions. As it turns out, the two models in Chapter 4 possess some of the qualities we are seeking.

Despite its weaknesses, the Rubinstein model remains a very solid model. It is not only of historical importance and has inspired much future research: it also serves as a benchmark in the models presented throughout the rest of the thesis, as the union's worst outcome is usually given by the Rubinstein share. It can therefore, as we shall see, be used to punish a deviating player.

The two approaches we describe in the following section are both models of perfect information, and it is not at all clear that such models support the desired qualities: if both the firm and the union have perfect information not only about the circumstances but also about each other's strategies, why would they not agree in the first period?

[3] "Surplus destruction" is precisely defined in Chapter 4.

CHAPTER 4

Strikes and surplus destruction

4.1 Introduction

We are now ready to discuss bargaining models that allow for the opportunity to delay time and destroy surplus. In this section we consider a profit or surplus denoted by Π that the union and firm has to split, and in this context the term surplus destruction refers to any action that directly or indirectly decreases the available surplus. Fernandez & Glazer (FG) [5] do not use the term surplus destruction, but they refer to strikes, which are destructive actions. However, Busch, Shi & Wen (BSW) [6] use the term surplus destruction.

We find it useful to distinguish between *temporary* and *permanent* surplus destruction.[1] We refer to surplus destruction as being temporary if it only inflicts temporary damage. An example of this is the FG model, where surplus reappears in each period. Even though the union has the power to entirely destroy the surplus, it reappears at full scale in the following period. In contrast, BSW introduce a model in which any destructive action by the union carries over into the next period. That is, if the union destroys one third of the surplus in the first period only two thirds is left in the second period.

The following sections motivate, introduce, analyze and offer critique of the FG and BSW models, but this requires understanding why we have chosen to on these two models. As mentioned in the critique of the Rubinstein model, it allows for neither inefficient equilibria nor real time delay. Both models we present support these features, although the underlying dynamics used to accomplish this are very different. As we will see, this can be used to support inefficient equilibria.

Lastly, considering what is understood by a real time delay is useful. It is a delay that persists even when the time between bargaining periods approaches 0; this is usually called the disappearance of bargaining friction. More formally, let Δ be the

[1] Chapter 5 provides additional details, where we introduce our model that emphasizes the difference between temporary and permanent surplus destruction. In defining our model, it also becomes clear that surplus destruction can be neither purely temporary or purely permanent.

time between bargaining periods and T the period in which agreement is reached. The existence of real time delay is equivalent to

$$\lim_{\Delta \to 0} T \cdot \Delta > 0.$$

4.2 Bargaining games with strikes

Fernandez & Glazer (FG) [5] were among the first to include a strike as an option for one player in a two-player bargaining game of alternating offers if either of the players has rejected a proposal. One year earlier, Haller & Holden [15] introduced a simpler model that contained the same idea of striking. However, FG are more rigorous in their model, which is why we only present their model in detail leaving a reference to Haller & Holden for the interested reader.

Earlier models of wage bargaining did not allow for agreement to occur in periods after the first one. To achieve agreement in later periods, which is often the case during wage bargaining as seen in real life, game theorists used tools of asymmetric information, which has the disadvantage of having strike only as a signalling device. FG, however, showed a way to extend Rubinstein's bargaining game with the possibility of a delayed agreement in a world of perfect information. This is an important step, since models of perfect information are often more accessible and convenient to solve than models of imperfect information, and strikes do occur as part of a strategy rather than as a pure signalling device. However, it is questionable whether perfect information in wage bargaining between a union and a firm is the most realistic way of describing the situation. Nevertheless, it is a useful tool for examining the dynamics occurring during strikes. As we will see, the model allows for multiple equilibria.

In the remainder of this part, we present the model presented by Fernandez & Glazer, the FG-model. Thereafter, we give an in-depth solution specifying some of the main results, including the existence of multiple efficient and inefficient equilibria. We comment on the finding and finally conclude.

4.2.1 The Fernandez & Glazer model

Consider a bargaining situation between two players, a union and a firm. They want to renegotiate the wage contract already existing between the firm and the union's workers.[2] A wage contract has the following form: the firm makes a constant profit per period of $\Pi \geq 0$ and the union's share at the beginning of the game—the old wage contract—is a share of the profit that we state here in absolute terms: that is, $w_0 \in [0, \Pi]$. Negotiations take place over discrete time periods $t \in \mathbb{N}$, and the union and the firm renegotiate the contract with wage offers $x_t \in [0, \Pi]$. In odd-numbered periods, the union makes a proposal and the firm needs to decide whether to accept (Y) or reject (N) the proposed contract. In even-numbered periods, the firm is the proposal-maker, and the union responds. Moreover, the union has the opportunity to strike "s" or not to strike "ns" whenever a player has rejected the proposal-maker's offer. If a player accepts the other player's offer, the game ends with the proposed wage contract. If neither of the players accepts an offer, the game may last an

[2]In the remainder of the thesis, we normalize the number of workers in the union to 1.

infinite number of periods. Lastly, let $s_i \in S_i$ be one of player i's strategies, where S_i is the set of all of player i's strategies.

A crucial difference from the Rubinstein bargaining model is that each player enjoys nonnegative utility in every period. Thus, the utility for the union at time t can be determined by

$$w_t = \begin{cases} 0, & \text{when there is a strike and no agreement in period } t \\ w_0, & \text{when there is no strike and no agreement in period } t \\ w, & \text{when agreement is reached.} \end{cases} \tag{4.1}$$

Keeping in mind that the firm receives $\Pi - w_0$ in every period in which the union does not strike and is still operating under the terms of the old wage contract, we can analogously define the firm's utility at time t by

$$v_t = \begin{cases} 0, & \text{when there is a strike and no agreement in period } t \\ \Pi - w_0, & \text{when there is no strike and no agreement in period } t \\ \Pi - w, & \text{when agreement is reached.} \end{cases} \tag{4.2}$$

In accordance to Rubinstein's bargaining model, the union and the firm face discount factors of $\delta_U \in (0,1)$ and $\delta_F \in (0,1)$, respectively.

The union and the firm clearly want to maximize their total utility. Keeping in mind that the utility for the union [the firm] at time t has a present value utility of $\delta_U^{t-1} w_t$ [$\delta_F^{t-1} v_t$], we can write the union's [firm's] total utility as $U_U = \sum_{t=1}^{\infty} \delta_U^{t-1} w_t$ [$U_F = \sum_{t=1}^{\infty} \delta_F^{t-1} v_t$]. Note that these sums are finite, as they are dominated by $\Pi \sum_{t=1}^{\infty} \delta_U^{t-1}$, which is a geometric series.

As we will show, this model has the important feature that it contains both multiple efficient and inefficient equilibria. Rational players are often thought of as playing in accordance with the Pareto postulate. However, in reality we, often see inefficient behavior that economists have difficulties in aligning with the rationality of the players involved. Nevertheless, with the proper strategy, inefficient equilibria are indeed part of a game played by a rational player. There is an opportunity to delay agreement, as seen in strike scenarios in the real world, which implies inefficiency. In the following section, we first focus attention on efficient equilibria and thereafter analyze the inefficient ones, as FG state.

4.2.2 Efficient subgame perfect equilibria

If the union's strategy is to strike in every period without agreement, we find ourselves in the original Rubinstein model as described in Chapter 2. Striking in every period makes the per-period payoffs become zero for both players, and the game is reduced to Rubinstein's bargaining game about a pie of size Π. To ease notation in the remainder of this thesis we define the following.

Definition 4.1. *For δ_U and δ_F both in $(0,1)$ a function $b : (0,1)^2 \to \mathbb{R}$ as follows*

$$b(\delta_U, \delta_F) := \frac{1-\delta_F}{1-\delta_U \delta_F}.$$

Note that when the union makes the first offer, the Rubinstein equilibrium found in (3.3) is given by $(b(\delta_U, \delta_F)\Pi, \delta_F b(\delta_F, \delta_U)\Pi)$ and that $b(\delta_U, \delta_F)\Pi + \delta_F b(\delta_F, \delta_U)\Pi = \Pi$. To find the Rubinstein equilibrium in the FG model, the two authors use a quite unconventional method. They are fixing a part of the strategy of the union to look for an SPE in the non-fixed part of the game. Thus, fixing the union's strategy to always strike whenever a wage offer is rejected reduces the game to the simple bargaining game from Chapter 3. Thus, the Rubinstein equilibrium is the unique outcome and the following lemma holds.

Lemma 4.2. *If the union's strategy is to strike in every period in which no agreement is reached, the bargaining game described above has a unique SPE with an agreement in the first period of the game. The SPE is a wage contract with an outcome for the union of*

$$\bar{w} = b(\delta_U, \delta_F)\Pi \tag{4.3}$$

when the union makes the first proposal: that is, the game starts in an odd-numbered period. Analogously, when the firm makes the first proposal—that is, the game starts in an even-numbered period—the outcome is

$$\bar{z} = \delta_U b(\delta_U, \delta_F)\Pi. \tag{4.4}$$

.

Considering the model, a possible wage contract to be generated as an SPE is the old wage contract w_0. This is especially interesting, since this would imply that the union could always choose to work for the old wage contract in equilibrium. Suppose the union is committed to the strategy that prescribes never to strike and to offer $x_t = w_0$ in every odd-numbered period. In even-numbered periods, in contrast, it replies to the firm's offer by accepting the wage contract whenever $x_t \geq w_0$ and rejecting it otherwise. We may state the unions strategy as

$$s_U' = \begin{cases} w_t = w_0, & \text{when } t = 1, 3, 5, \ldots \\ ns, & \text{for all } t \in \mathbb{N} \\ Y, & \text{when } t = 2, 4, 6, \ldots \text{ and the firm's offer is } x_t \geq w_0 \\ N, & \text{when } t = 2, 4, 6, \ldots \text{ and the firm's offer is } x_t < w_0 \end{cases}$$

Now consider the firm's strategy. The firm will simply offer $x_t = w_0$ in every even-numbered period and accept an offer by the union in an odd-numbered period whenever $x_t \leq w_0$ and reject it otherwise. The firm's strategy may be stated as

$$s_F' = \begin{cases} w_t = w_0, & \text{when } t = 2, 4, 6, \ldots \\ Y, & \text{when } t = 1, 3, 5, \ldots \text{ and the union's offer is } x_t \leq w_0 \\ N, & \text{when } t = 1, 3, 5, \ldots \text{ and the union's offer is } x_t > w_0 \end{cases}$$

Lemma 4.3. *The old wage contract w_0 can be generated as an SPE with an agreement to w_0 made in the first period if the union is committed to play s_U' and the firm is committed to play s_F'.*

Proof. The result is proven by construction using the strategies s_U' and s_F'. Using the one-deviation property (Corollary 2.8), showing that these strategies are subgame perfect is easy. □

The old wage contract is not the best wage contract that the union can achieve. However, since the union, through previous negotiations, is guaranteed a wage contract w_0 and a lower wage contract can never be generated as an SPE (see proof of Theorem 4.6), w_0 can be called the minimum contract to be supported as an SPE.

Having found the minimum wage contract that the union can achieve accompanied by a quite reserved striking strategy (never to strike), we now restrain the conditions under which the original Rubinstein equilibrium can be achieved. Given

that the old wage contract is a kind of "guaranteed" contract to the union, the Rubinstein equilibrium outcome must clearly yield as least as good a wage contract compared with w_0. Given equations (4.3) and (4.4), this applies when $w_0 \leq \delta_U \bar{z}$.
Lemma 4.4. *If $w_0 \leq \delta_U \bar{z}$, an SPE exists with an agreement of $\bar{w}$ reached in the first period.*

Proof. Consider the strategies

$$s_U = \begin{cases} w_t = \bar{w}, & \text{when } t = 1,3,5,\ldots \\ s, & \text{when } t = 1,3,5,\ldots \text{ when the firm has rejected } w_t = \bar{w} \\ ns, & \text{for all } t = 2,4,6,\ldots \\ Y, & \text{when } t = 2,4,6,\ldots \text{ and the firm's offer is } x_t \geq \bar{w} \\ N, & \text{when } t = 2,4,6,\ldots \text{ and the firm's offer is } x_t < \bar{w} \end{cases}$$

and

$$s_F = \begin{cases} w_t = \bar{w}, & \text{when } t = 2,4,6,\ldots \\ Y, & \text{when } t = 1,3,5,\ldots \text{ and the union's offer is } x_t \leq \bar{w} \\ N, & \text{when } t = 1,3,5,\ldots \text{ and the union's offer is } x_t > \bar{w}. \end{cases}$$

If $w_0 \leq \delta_U \bar{z}$—that is, $\bar{w}$ is at least as good as w_0—the strategies s_U and s_F generate an SPE. This can easily be checked. If $w_0 > \bar{w}$, the union would be better off to stay with the old contract, which is why $w_0 \leq \delta_U \bar{z}$ is a necessary condition for $\bar{w}$ to be an SPE. □

Case 1: Strike of the German Engine Drivers' Union in Germany 2007–2008
This case motivates the findings of the FG model empirically. The German Engine Drivers' Union (GDL) was striking to achieve a higher wage contract. In June 2008, the first offer by the German National Railways (DB) of an increase of 4% was rejected. The firm then rejected the Union's following offer. The Union used the situation to call for a temporary strike, whereafter the negotiations continued. In the following 9 months the game was repeated again and again: GDL constantly rejected DB's wage contracts, DB constantly rejected GDL's wage contracts and the latter rejections were followed by a strike. This strategy of striking after the union's own proposal was rejected is clearly consistent with the strategies generating efficient equilibria in the FG model. This kind of strike was supposed to increase the bargaining power of the Union. In March 2008, DB and GDL finally agreed on a wage increase of 11% following 9 months of negotiation. This result provides empirical evidence supporting the Union's increased bargaining power in its equilibrium strategy in the FG model.

We now turn our attention to finding the union's highest possible contract in an SPE. We note that the union is guaranteed w_0 as given by the old wage contract. Thus, we imagine the bargaining situation between the union and the firm to be about the remaining share of the profit: $\Pi - w_0$. Intuitively, we remark that the greatest share the union could achieve is the Rubinstein share: that is, $b(\delta_U, \delta_F)(\Pi - w_0)$. Following this we define $w' = w_0 + b(\delta_U, \delta_F)(\Pi - w_0)$ and $z' = w_0 + \delta_U b(\delta_U, \delta_F)(\Pi - w_0)$. Using algebra, these results can be written as

$$w' = \bar{w} + \delta_F w_0 b(\delta_F, \delta_U) \tag{4.5}$$

and

$$z' = \bar{z} + w_0 b(\delta_F, \delta_U). \tag{4.6}$$

For this w' to be the maximum wage contract, we again must have it yield at least as much payoff as the old wage contract. Thus, it must hold that $w_0 \leq \delta_U z'$.[3]

Lemma 4.5. *The union's maximum wage contract that can be generated as an SPE is w', with an agreement of*

$$w' = \bar{w} + \delta_F w_0 b(\delta_F, \delta_U)$$

reached in the first period if $w_0 \leq \delta_U z'$.

Proof. The proof is divided into two parts. In the first part, we show the equilibrium strategies that support w' as an SPE. The second part shows that w' is the maximum wage contract the union can achieve.

Consider the following strategy of the union,

$$s_U^* = \begin{cases} w_t = w', & \text{when } t = 1, 3, 5, \ldots \\ s, & \text{when } t = 1, 3, 5, \ldots \text{ when the firm has rejected } w_t = w' \\ ns, & \text{for all } t = 2, 4, 6, \ldots \\ Y, & \text{when } t = 2, 4, 6, \ldots \text{ and the firm's offer is } x_t \geq w' \\ N, & \text{when } t = 2, 4, 6, \ldots \text{ and the firm's offer is } x_t < w' \end{cases}$$

and of the firm

$$s_F^* = \begin{cases} w_t = w', & \text{when } t = 2, 4, 6, \ldots \\ Y, & \text{when } t = 1, 3, 5, \ldots \text{ and the union's offer is } x_t \leq w' \\ N, & \text{when } t = 1, 3, 5, \ldots \text{ and the union's offer is } x_t > w'. \end{cases}$$

If, however, the union at some point deviates from taking an action in s_U^*, the strategy calls for both players to play the equilibrium of Lemma 4.3. We call this part of the strategy the "deviation property". This "punishment" of deviation is supposed to stress the credibility of the players' strategies. The strategy profile (s_U^*, s_F^*) plus the deviation property is an SPE.

For the second part of the proof, see the proof to Lemma 4 in FG [5]. □

Behind the strategies supporting w' lies simple intuition. By striking only after the rejection of its own proposal, the union makes it more costly for the firm not to accept its offer. This means that, besides the discount factor, the firm now also faces the situation of losing a share of its profit. This automatically puts the union in a better negotiating position since it always chooses to work based on the firm's offer and thus earns the old wage of w_0.

After the minimum and maximum contracts that can be supported as an SPE have been found, the set of all wage contracts that can be supported as an efficient SPE can be derived. That is, if w' can be an equilibrium contract, so can any contract between the minimum and maximum contract.

Theorem 4.6. *Any wage contract w in which $w_0 \leq w \leq w'$ can be supported as an SPE with agreement in the first period if $w_0 \leq \delta_u z'$.*

Proof. FG's proof of Theorem 1 in [5] proves this theorem following the equilibrium strategies. □

[3] Bolt shows in a comment on the FG model [9] an additional requirement on the discount factors of both the firm and the union that must hold to verify w' as the maximum wage contract. We give a short overview of his finding below.

The Bolt discussion In a response to FG, Bolt [9] showed that the analysis of FG does not exactly hold as stated. He shows that the maximum wage contract as stated by FG only exists when $\delta_F \geq \delta_U$. For all $\delta_F < \delta_U$, the firm is not interested in allowing the maximum wage contract to pass but is instead committed to following the no-concession strategy.[4] The no-concession strategy results in a payoff to the firm that equals the discounted value of the disagreement payoffs: that is, $(1-\delta_F)\sum_{t=1}^{\infty}\delta_F^{t-1}v_t$.[5] Since the union's strategy tells it to strike in every odd-numbered period after the firm has rejected the union's offer (which it will do according to the no-concession strategy), v_t will be zero for all odd t. Thus, we can do some algebra on the no-concession payoff of the firm using some unusual denotation for the sum.

$$\begin{aligned}(1-\delta_F)\sum_{t=1}^{\infty}\delta_F^{t-1}v_t &= (1-\delta_F)(\delta_F+\delta_F^3+...)(F-w_0)\\ &= (1-\delta_F)\delta_F(1+\delta_F^2+...)(F-w_0)\end{aligned}$$

We see that the last sum is a geometric series with the argument δ_F^2. Thus, we get

$$\begin{aligned}(1-\delta_F)\delta_F(1+\delta_F^2+...)(F-w_0) &= (1-\delta_F)\delta_F\frac{1}{1-\delta_F^2}(F-w_0)\\ &= (1-\delta_F)\delta_F\frac{1}{(1-\delta_F)(1+\delta_F)}(F-w_0)\\ &= \delta_F\frac{1}{1+\delta_F}(F-w_0)\end{aligned}$$

We can now analyze the cases when the firm prefers the no-concession outcome to the maximum wage contract. This is when

$$\delta_F\frac{1}{1+\delta_F}(F-w_0) > F-w'$$

Using Equation (4.5), we see that $F-w' = \delta_F\frac{1-\delta_U}{1-\delta_U\delta_F}(F-w_0)$. Thus, the above inequality only holds when $\delta_F < \delta_U$, proving the above statement that FG's result only holds when $\delta_F \geq \delta_U$.

Bolt comes up with an alternative strategy that generates an SPE if $\delta_F < \delta_U$. Define $\breve{w} = \frac{F+\delta_F w_0}{1+\delta_F}$

$$\breve{s}_U = \begin{cases} w_t = \breve{w}, & \text{when } t = 1,3,5,\dots\\ s, & \text{when } t = 1,3,5,\dots \text{ when the firm has rejected } w_t = \breve{w}\\ ns, & \text{for all } t = 2,4,6,\dots\\ Y, & \text{when } t = 2,4,6,\dots \text{ and the firm's offer is } x_t \geq \breve{w}\\ N, & \text{when } t = 2,4,6,\dots \text{ and the firm's offer is } x_t < \breve{w}\end{cases}$$

and of the firm

$$\breve{s}_F = \begin{cases} w_t = 0, & \text{when } t = 2,4,6,\dots\\ Y, & \text{when } t = 1,3,5,\dots \text{ and the union's offer is } x_t \leq \breve{w}\\ N, & \text{when } t = 1,3,5,\dots \text{ and the union's offer is } x_t > \breve{w}.\end{cases}$$

Here as well, the deviation property holds that, when the union deviates, the strategy tells both players to play according to Lemma 4.3. For a proof of this result, consult Bolt [9].

[4] This strategy tells the firm never to accept any of the offers suggested by the union in odd-numbered periods and to make unacceptable wage offers (such as $w_t = 0$) in even-numbered periods.

[5] Note that this payoff is normalized. Moreover, recall v_t from Equation (4.2).

4.2.3 Inefficient subgame perfect equilibria

We now show that inefficient SPE exist in FG's strike model of perfect information. Inefficiency is implied by equilibria in which an agreement is not reached in the first period, but rather period $T+1$ for some T. One wonders why the involved parties do not agree on the same share in the first period, thus avoiding inefficiency, but as we show in this section, an even worse outcome blocks an agreement in the first period. More generally, the outcome in any period prior to the period $T+1$ will be worse. Such an equilibrium can occur through a continuous strike for T periods.[6]

Suppose that the firm is seeking a wage contract $\hat{w}$ after the union has been on strike for T consecutive periods. In order to make it rational for the union to be interested in striking for T periods, the wage contract agreed upon in period $T+1$ must satisfy $\delta_U^T \hat{w} \geq w_0$: that is, its present value in the first period must be at least as good as choosing the original wage contract in the first period. On the other hand, it must also hold that the firm prefers suffering for T periods to achieving an immediate agreement of $\bar{z}$. This idea is motivated by the equilibrium strategies described below. As with efficient equilibria, the strategies also include a deviation property: that is, if the firm would deviate from the below stated equilibrium strategy, players are forced to play the equilibrium in Lemma 4.4. Now, the firm must prefer to suffer from strikes for T periods to achieving $\bar{z}$ immediately. Surely, the firm would deviate in the first round—that is reject the union's offer—if the from the union proposed contract would not satisfy this to achieve $\bar{z}$ which is assured through the deviation property. Mathematically, this is expressed as $\Pi - \bar{z} \leq \delta_F^{T-1}(\Pi - \hat{w})$, which can be rearranged to $\hat{w} \leq (1-\delta_F^{1-T})\Pi + \delta_F^{1-T}\bar{z}$. Thus, we have found a lower and an upper bound for $\hat{w}$ and formalize it in the following theorem.

Theorem 4.7. *There is a subgame perfect equilibrium with an agreement of $\hat{w}$ in period $T+1$ after T periods of strikes by the union if*

$$\delta_U^{-T} w_0 \leq \hat{w} \leq (1-\delta_F^{1-T})\Pi + \delta_F^{1-T}\bar{z}. \tag{4.7}$$

Proof. For a formal proof of the theorem, consult FG's appendix [5]. □

In the following section we describe the strategy used to create inefficient equilibria. As mentioned above, the union will strike in every period $t \in \{0, \dots, T\}$. To accomplish this, the union needs to make unreasonably high wage demands in the first T periods to ensure that the firm rejects them and thereby gives the union an opportunity to strike. These wage offers can be, for example, x_t's close to Π or any demand that satisfies $x_t > (1-\delta_F^{1-T})\Pi + \delta_F^{1-T}\bar{z}$. From period $t = T+1$ and onwards, the union will demand $x_t = \hat{w}$ for $t = T+1, T+3, T+5\dots$ if $T+1$ is odd (consequently, for $t = T+2, T+4, \dots$, it will accept wage offers by the firm if and only if $y_t \geq \hat{w}$). On the other hand, if $T+1$ is even, the union will accept offers $x_t \geq \hat{w}$ in $t = T+1, T+3, \dots$ and propose $x_t = \hat{w}$ in $t = T+2, T+4, \dots$ In both situations we impose the additional strike criterion that the union would strike again whenever the firm has rejected its proposal. The strategy of the union can be summarized as follows.

[6] FG argue that their results also hold in the situation of interrupted strikes.

$$\hat{s}_u = \begin{cases} x_t & \begin{cases} > (1-\delta_f^{1-T})\Pi + \delta_f^{1-T}\bar{z}, & \text{if } t \in \{1,\dots,T\} \\ = \hat{w}, & \text{when } t = T+1, T+3, \dots \text{ and } T+1 \text{ is odd} \\ = \hat{w}, & \text{when } t = T+2, T+4, \dots \text{ and } T+1 \text{ is even} \end{cases} \\ \\ s, & \text{when } t \in \{1,\dots,T\} \\ & \text{when } t = T+1, T+3, \dots\text{, } T+1 \text{ even, and the firm has rejected } x_t = \hat{w} \\ & \text{when } t = T+2, T+4, \dots\text{, } T+1 \text{ odd, and the firm has rejected } x_t = \hat{w} \\ \\ ns, & \text{for all } t = T+2, T+4, \dots\text{, } T+1 \text{ odd} \\ & \text{for all } t = T+1, T+3, \dots\text{, } T+1 \text{ even} \\ \\ Y, & \text{when } t = T+2, T+4, \dots\text{, for } T+1 \text{ odd and } y_t \geq \hat{w} \\ & \text{when } t = T+1, T+3, \dots\text{, for } T+1 \text{ even and } y_t \geq \hat{w} \\ \\ N, & \text{when } t \in \{1,\dots,T\} \\ & \text{when } t = T+2, T+4, \dots\text{, for } T+1 \text{ odd and } y_t < \hat{w} \\ & \text{when } t = T+1, T+3, \dots\text{, for } T+1 \text{ even and } y_t < \hat{w} \end{cases}$$

During the first T periods the firm offers a non-serious low wage contract, such as $y_t = w_0$, being well aware that the union will reject it. Moreover, it never accepts a wage contract proposed by the union during the first T periods. However, from period $T+1$, the firm's strategy is to accept any wage offer $x_t \leq \hat{w}$ for $t = T+1, T+3, \dots$ if $T+1$ is odd and for $t = T+2, T+4, \dots$ if $T+1$ is even. Moreover, the firm will now propose a wage contract $y_t = \hat{w}$ for $t = T+2, T+4, \dots$ if $T+1$ is odd, and for $t = T+1, T+3, \dots$ if $T+1$ is even. The firm's strategy can be summarized as follows.

$$\hat{s}_f = \begin{cases} y_t & \begin{cases} = w_0, & \text{if } t \in \{1,\dots,T\} \\ = \hat{w}, & \text{when } t = T+2, T+4, \dots \text{ and } T+1 \text{ is odd} \\ = \hat{w}, & \text{when } t = T+1, T+3, \dots \text{ and } T+1 \text{ is even} \end{cases} \\ \\ Y, & \text{when } t = T+1, T+3, \dots\text{, for } T+1 \text{ odd and } x_t \leq \hat{w} \\ & \text{when } t = T+2, T+4, \dots\text{, for } T+1 \text{ even and } x_t \leq \hat{w} \\ \\ N, & \text{when } t \in \{1,\dots,T\} \\ & \text{when } t = T+1, T+3, \dots\text{, for } T+1 \text{ odd and } x_t > \hat{w} \\ & \text{when } t = T+2, T+4, \dots\text{, for } T+1 \text{ even and } x_t > \hat{w} \end{cases}$$

A strictly better payoff could be reached if the union and the firm agreed to the wage contract $\hat{w}$ in the first period rather than in period $T+1$. Actually, such an agreement (and others, such as when $\hat{w}$ is agreed on in periods prior to $T+1$) Pareto dominates this inefficient equilibrium. Deviations by either player are mutually blocked by the following property included in the equilibrium strategies. If the union at some point decides to deviate from its strategy by, for example either suggesting $\hat{w}$ in an earlier period than $T+1$ or by not striking at some $t \in \{1,\dots,T\}$, then the strategy profile punishes the union's deviation by having both players play the equilibrium strategies described in Lemma 4.3. Thus, this threat keeps the union from deviating since it will end up with a weakly worse payoff than if the players agreed on $\hat{w}$ in $T+1$. On the other hand, should the firm deviate at some point by offering a wage contract in $t \in \{1,\dots,T\}$ that the union strictly prefers to the agreement of $\hat{w}$ at $T+1$, the strategy requires both players to play the strategy described in Lemma 4.4, which the union weakly prefers to an agreement of $\hat{w}$, while it is worse for the firm. Thus, the punishment property keeps both players from deviating from their equilibrium strategy.

We have now presented the important result that FG were among the first to state: the possibility of delayed agreement in a bargaining game with perfect infor-

Case 2: Strike by the Danish Nurses' Organization in Denmark in 2008
On April 16, 2008, the Danish Nurses' Organization began a strike to increase their salary. Briefly, most public employees had negotiated a wage increase of 12.8% over three years, but the nurses indicated that they were already underpaid, demanded a higher increase of 15% and called a continuous strike until agreement was reached.

The FG model supports these strategies, especially the inefficient equilibria. The increase of 12.8% in the rest of the public sector can be considered an inadequate wage contract offered by Denmark's municipalities and administrative regions, the employers of the nurses. On the other hand, the 15% wage increase demand by Danish Nurses' Organization was set too high. Thus, given the FG model, we find ourselves in the $T-1$ periods of a continuous strike.

On June 25, 2008, the municipalities and administrative regions and the Danish Nurses' Organization agreed on a wage increase of 13.3%. The continuous strike lasted for more than two months, and the outcome can be seen as the equilibrium wage contract. An increase of 0.5% might not seem like a very great result for the union. However, counting the free public relations for the nurses in the mass media, the payoff to the union in immaterial terms was even higher. They pointed out a possible problem in Denmark's health system: underpayment of nurses.

mation. In the next section we comment on and discuss the FG model, its assumptions and its reference to reality.

4.2.4 Comments and conclusion on the Fernandez and Glazer model

The introduction of the option to strike and thereby to destroy surplus in a game with complete information is a valuable addition to the theory of bargaining. It enables one player to be in a better bargaining position and thereby leaves the player with a higher payoff. Another feature of the FG model is the existence of multiple equilibria achieved by the possibility of destroying surplus. There are two groups of equilibria: efficient and inefficient ones. For the efficient equilibria, agreement is always reached in the first period. The union only threatens to strike after the firm has rejected the union's offer. A rejection of the firm's offer is not followed by a strike. This makes it more costly to reject the union's offer than the firm's offer which results in a better bargaining position for the union. Thus, in efficient equilibria, strikes are never actually penalized. Inefficient equilibria, in contrast, include periods of actual strikes. These strikes are motivated by the fact that the union makes its wage claim believable by striking for some time and then changes to and follows a strategy similar to those used in efficient equilibria (that is, never to strike after rejecting the firm's offer and always to strike after the firm rejects the union's offer). The question arising, however, is whether the FG model makes relevant assumptions.

The assumption of perfect information in a bargaining world seems quite optimistic. For a huge firm with many employees, does it seem realistic to assume that the profitability of the firm is public information to the union as well? Personal compensation as well as after-tax profit are certainly part of a company's public accounts. However, it seems questionable to use these numbers as a basis for wage bargaining. Other factors should be taken into consideration, such as a firm's roadmap and investment plan, its market power and its overall performance. A change in the salaries a company pays may even affect the company's profit directly or indirectly (for example, as part of necessary investments). Suddenly, the bargaining situation appears more complicated and is indeed in question if the union accounts for other variables than the pure profit. Speaking of a bargaining situation with perfect information when describing a real-life situation is therefore a bold assumption. However, it is not completely unrealistic to assume perfect information. Consider the nurses that went on strike in Denmark or the engine drivers in Germany. Both the nurses and the engine drivers are paid by public-sector institution. With the public-sector institution being the firm in the FG model, it seems acceptable to assume that the parameters of the game are information known to both parties in the game since here the "firm's" budget indeed can be seen as public to both the the public institution and the union.

Another debatable fact occurring in the FG model is the real time delay. How believable and effective are strikes if the length of the time periods converges towards zero? FG avoid addressing this issue in depth and explain that strikes in their model do not serve as a signalling device (as is often the case with incomplete information models). This is a necessary tool of the union's strategy because not striking would mean a lower wage for the union (since without the threat of striking the union can only achieve the old wage contract w_0). However, an interrupted strike seems unrealistic when time periods converge towards zero: that is, striking when the union's offer is rejected and not striking when the firm's offer is rejected. Setting up and coordinating a strike may take longer than reaching the next period in bargaining when time periods are arbitrarily small. Thus, for the efficient equilibria, FG may have difficulty in arguing for their form of surplus destruction. On the other hand, strikes seem reasonable considering the inefficient equilibria, because here you can build a strategy with arbitrarily many strike periods as long as inequality (4.7) continues to hold for some wage contract $\hat{w}$ to be agreed on.

The FG model as presented above is a model of temporary surplus destruction. Two cornerstones of the model need closer analysis. Firstly, FG argue for the situation of reappearing surplus: that is, the original profit appears to be the same in a period following a period of disagreement. This is quite strong assumption. Consider the engine drivers' strike. During a railway strike , people have to look for transport alternatives, such as commuting. When the trains stand still, they may start carpooling with some of their colleagues. Doing so, they might find out that carpooling is much more entertaining, since you may chat with colleagues you do not normally see during work in a more comfortable environment than an overcrowded commuter train. This may result in a continuous switch from commuter trains to carpooling. Even though engine drivers continue to work, after some time a share of the profit might thus have been lost. FG do not capture this situation of non-reappearing surplus. Secondly, the assumption that a strike amounts to complete surplus destruction, what we call FG's all-or-nothing approach, is debatable. In many situations, a strike may not destroy all surplus. Consider the nurses' strike in Denmark. It is fair to think of some basic supply of health care in case of emergency thus, securing some basic surplus.

In conclusion, the FG model captures some valuable aspects, of which the multiplicity of equilibria and the existence of inefficient equilibria are the most appreciable. That is, FG, in the framework of perfect information, have found an answer to the question of, why rational players might have inefficient behavior. However, their model leaves room for improvement, including the possibility of partial surplus destruction and the lasting effects of surplus destruction with some share not reappearing in subsequent periods. In the remainder of this chapter we examine the model by Busch, Shi and Wen [6] that incorporates partial surplus destruction, though, without reappearing surplus.

4.3 Permanent surplus destruction

The previous sections showed how the Rubinstein bargaining model from Chapter could be expanded3 with temporary surplus destruction through strikes and hence reach multiple and inefficient equilibria. Another possible generalization is to introduce the concept of permanent surplus destruction into the model. The basic idea is that one of the players—in this section the union—has the power to destroy some of the surplus, and once this has been destroyed the surplus is lost permanently. This contrasts with temporary surplus destruction in the FG model, in which a strike destroys the entire surplus, but it reappears in the following period and negotiation begins anew.

Eliminating the reappearing surplus feature of the model may seem like an unattractive simplification, as most firms would continue to have some revenue even during a strike. BSW motivate their model with the fact that it is often not the forgone income during a strike but rather the lasting effects on future profit that affect the overall outcome of the firm the most. As examples, BSW provide, the possibility that consumers exploring new ways to cover their needs during the strike stumble upon a better product or service and keep using it even after the strike ends. Further, material and machines might permanently lose value due to lack of maintenance. Finally, the (illegal) possibility of sabotage is considered. BSW remark that expanding the model to take sabotage into account would not be too difficult if we consider the union to be risk-neutral.

The model about to be described offers a continuum of possible levels of surplus destruction, in contrast to the all-or-nothing approach of the FG model. This feature is certainly very attractive for numerous reasons. First, myriad situations can be thought of in which all surplus cannot be destroyed. For example many firms continue to have cash flow and therefore profit although the workers are on strike. In many situations, the entire working staff—for instance police officers and fire fighters—cannot strike simultaneously for various reasons. We discuss this in depth in Chapter 5.

The BSW model shows that surplus destruction by the union effectively amounts to decreasing the firm's discount factor; a result already proven by Avery & Zemsky (AZ) [7]. Intuitively, this is not clear at all. The discount factor is affected by the firm's preferences and even exogenous factors such as interest rates, both of which the union cannot influence. However, the BSW approach provides a justification of this interpretation and provides an intuitive way of understanding this fact as a consequence of Proposition 4.8.

The presentation that follows uses material, ideas and perspectives from AZ [7] as well as Manzini [8] but is primarily based on BSW [6]. For the interested reader, AZ [7] briefly describes and discuses possible generalizations such as opting out, N person bargaining, the possibility of retracting offers and continuous time models.

4.3.1 The Busch, Shi and Wen model

As is customary in this thesis, we consider discrete time periods $t \in \mathbb{N}$ and denote the union's share of the surplus $\Pi > 0$ by $x \in [0, \Pi]$. BSW normalize the profit to 1, but we believe that not doing so makes the results become more transparent and at least easier to generalize. Both parties can either accept, (Y), or reject, (N). Let δ_U and δ_F, both in $(0, 1)$, be the discount factors for the firm and the union, respectively. The possibility of surplus destruction is introduced through $\gamma \in [0, 1]$. Letting π_t denote the available surplus in period t, we let

$$\pi_{t+1} = \gamma\pi_t, \quad \pi_1 = \Pi.$$

Supported by our intuition, we call $1 - \gamma$ the level of (surplus) destruction. Note that in the model only the union can destroy surplus. An agreement in period T is denoted by $\Theta(T) := (\gamma_1, \ldots, \gamma_{T-1}, x)$, where $1 - \gamma_t$ is the union's level of destruction throughout the periods 1 to $T-1$ and x is the offer finally accepted. For an agreement $(x, \Pi - x)$ in period T, $\Theta(T)$, the union receives

$$x\Pi \prod_{t=1}^{T-1} \gamma_t \quad \text{and hence has utility} \quad \delta_U^{T-1} x\Pi \prod_{t=1}^{T-1} \gamma_t,$$

whereas the firm receives

$$(1 - x)\Pi \prod_{t=1}^{T-1} \gamma_t \quad \text{and hence has utility} \quad \delta_F^{T-1}(1 - x)\Pi \prod_{t=1}^{T-1} \gamma_t.$$

Should union and firm continue to disagree, we can interpret this as $T \to \infty$ and hence the utility for both of them approaches 0. We need to describe the set of histories, $H_k(t)$. The basic history, $h_1(t)$, is the one consisting of the $t-1$ offers, the rejections of each of these and $t - 1$ levels of destructions: that is, $1 - \gamma \in [0, 1]$:

$$h_1(t) := (x_1, N, 1 - \gamma_1, \ldots, x_{t-1}, N, 1 - \gamma_{t-1}) \in H_1(t), \text{ where}$$
$$H_1(t) = \Big([0, 1] \times \{N\} \times [0, 1]\Big)^{t-1} \text{ if } t \geq 2 \text{ and } H_1(1) = \emptyset.$$

Further, we have the histories in which a new offer has been made

$$h_2(t) := (h_1(t), x_t) \in H_2(t) := H_1(t) \times [0, 1].$$

Finally, the last kind of history is a rejection of $h_2(t)$: that is,

$$h_3(t) := (h_2(t), N) \in H_3(t) := H_2(t) \times \{N\}.$$

Lastly, we need to describe the strategy profile (f_U, f_F), which is a map from the set of histories into the set of appropriate actions

$$f_U : \bigcup_{t=1}^{\infty} [H_1(t) \cup H_2(t) \cup H_3(t)] \to [0, 1] \cup \{Y, N, Q\}$$

introducing Q, which stands for "quiet" and in which f_F maps between the same sets. For odd t, we hence get

$$\begin{aligned} &f_U|_{H_1(t)} \in [0, 1], \quad &&f_U|_{H_2(t)} = Q, \quad &&f_U|_{H_3(t)} \in [0, 1], \\ &f_F|_{H_1(t)} = Q, \quad &&f_F|_{H_2(t)} \in \{Y, N\}, \quad &&f_F|_{H_3(t)} = Q, \end{aligned}$$

and for even t

$$\begin{array}{lll} f_U|_{H_1(t)} = Q, & f_U|_{H_2(t)} \in \{Y, N\}, & f_U|_{H_3(t)} \in [0,1], \\ f_F|_{H_1(t)} \in [0,1], & f_F|_{H_2(t)} = Q, & f_F|_{H_3(t)} = Q. \end{array}$$

Note the asymmetry arising from the fact that only the union can destroy surplus after histories in $H_3(t)$. This completes the description of the model considered for surplus destruction, and henceforth we use $\Gamma_U(\delta_U, \delta_F)$ and $\Gamma_F(\delta_U, \delta_F)$ to denote the game in which the union and the firm, respectively, make the first offer.

4.3.2 Supported equilibria

This section describes the equilibria supported by the BSW model. We show that the Rubinstein equilibrium is a special case occurring when no surplus is destroyed: that is $\gamma = 1$. When the union is exercising (credible) surplus destruction or threatens to do so, this can lead to either a larger share for the union agreed on in the first period or a larger share when agreement is reached in a later period. The first case is clearly an efficient equilibrium, whereas the latter is inefficient. Theorem 4.10 gives the bounds of the supported equilibria. Finally, we derive the main result of this section, Theorem 4.11, which states the existence of inefficient equilibria and characterizes of them.

First, a simple proposition describes the efficient equilibria when surplus destruction is modelled.

Proposition 4.8. *For all $(\delta_U, \delta_F) \in (0,1)^2$ and for $\gamma \in [0,1]$ if*

$$\gamma b(\delta_U, \gamma\delta_F) \geq b(\delta_U, \delta_F) \tag{4.8}$$

is satisfied then $\Gamma_U(\delta_U, \delta_F)$ has an efficient SPE reached in the first period with share vector

$$(b(\delta_U, \gamma\delta_F), \gamma\delta_F b(\gamma\delta_F, \delta_U)). \tag{4.9}$$

Further, if (4.8) is fulfilled the union will destroy $1-\gamma$ after the firm has rejected its offer, while it will destroy 0 otherwise.

If $\gamma = 1$—no surplus is destroyed—equation (4.8) is trivially true and we return to the Rubinstein equilibrium agreed on in the first period. If $\gamma < 1$, surplus is destroyed and this is equivalent to changing the firm's share from

$$\delta_F b(\delta_F, \delta_U) \quad \text{into} \quad \gamma\delta_F b(\gamma\delta_F, \delta_U),$$

which again is amounts to changing the firm's discounting factor from δ_F to $\gamma\delta_F$. This is precisely AZ's result in [7] and means that the union has made the firm more impatient. As we argued in Chapter 3, this increased impatience benefits the union but hurts the firm. Although AZ merely provide motivation for the power of the union to change the firm's discounting factor the more abstract BSW approach offers a mathematical justification. We turn to the proof of Proposition 4.8.

Proof. Due to the stationarity of the game it suffices to consider the strategies for any odd and even period. In an odd period let the union propose $b(\delta_U, \gamma\delta_F)$, the firm reject if and only if the union demands more and the union destroys $1-\gamma$ if rejected by the firm. In an even period, let the firm propose $\delta_U b(\delta_U, \gamma\delta_F)$, the union rejects if and only if the firm offers less and let the union destroy $\gamma' = 1$ if it rejects the firm's offer. We now introduce trigger strategies, which are essential to proof: if the union at any time deviates from its level of surplus destruction then the Rubinstein equilibrium strategy is followed from the following period. This is known to be SPE. Thus, to check subgame perfection, it suffices to check the case in which the union deviates in the level of surplus destruction. We consider the odd and the even periods separately.

Odd period: if the union deviates by playing $\hat{\gamma} \neq \gamma$, the firm uses the trigger strategy and thus the union ends up receiving the share $b(\delta_U, \delta_F)$ in the following period. But as surplus has been destroyed there is only $\hat{\gamma}\Pi$ left and it further needs to be discounted by δ_U. Thus, the utility of the firm is $\delta_U \gamma' b(\delta_U, \delta_F)$. If the union does not deviate, it receives $\gamma \delta_U b(\delta_U, \gamma\delta_F)$. If no profitable deviation exists, it must be

$$\delta_U \gamma' b(\delta_U, \delta_F) \leq \delta_U \gamma b(\delta_U, \gamma\delta_F),$$

which holds by assumption (4.8) as $\gamma' \in [0, 1]$.

Even period: analogously to the odd period, for the union to play $\gamma' = 1$ there must not be a profitable deviation $\hat{\gamma}$. This is equivalent to

$$\delta_U \hat{\gamma} b(\delta_U, \delta_F) \leq \delta_U \gamma' b(\delta_U, \gamma\delta_F) = \delta_U b(\delta_U, \gamma\delta_F),$$

which obviously holds as $\partial b / \partial \delta_F < 0$. □

The proof of Proposition 4.8 has several important features. First, it introduces the idea of trigger strategies, which we use when proving results in our own model. Further it establishes the very important result that the union will never destroy surplus after a rejecting of the firm's offer. We have already offered an intuitive understanding of this as part of the FG model. The important thing to note is that, although AZ [7] exclude from their model the possibility of destroying surplus after the union rejects the firm's offer, BSW prove that this can never be profitable for the union. Lastly, we remark that stationarity of the preferences implies that Proposition 4.8 states that the union will always destroy at the same level in odd and even periods, respectively.

It is clear that

$$\frac{\partial(\delta_U, \gamma\delta_F)}{\partial \gamma} < 0. \tag{4.10}$$

As $b(\delta_U, \gamma\delta_F)$ is the unions share, equation (4.10) implies that the higher the level of destruction the higher the union's share will be. Nevertheless, the union cannot destroy the surplus at will, as it has to be credible. Whenever the union destroys part of the surplus it gains a larger share but of a smaller total. The union thus has to balance their level of destruction according to whether they want a larger part of a smaller surplus or a smaller part of a larger surplus. This balance requirement is formalized by equation (4.8). As shown in the proof of Proposition 4.8, whenever (4.8) holds, the threat to destroy surplus is credible. Hence the union's best choice of γ, which we denote by γ^*, is the smallest $\gamma \in [0, 1]$ for which (4.8) holds. But so far it is not at all clear that any pairs of (δ_U, δ_F) actually exists for which we can find a γ such that (4.8) is fulfilled. Fortunately, this turns out to be the case and the following section analyzes this. We must have

$$\gamma b(\delta_U, \gamma\delta_F) \geq b(\delta_U, \delta_F)$$

for $\gamma \in [0, 1]$, implying that

$$\gamma \geq \frac{b(\delta_U, \delta_F)}{\delta_F} = \frac{1-\delta_F}{\delta_F(1-\delta_F\delta_U)} =: f(\delta_U, \delta_F).$$

Whenever f is larger than 1, the union will just pick $\gamma = 1$ which, as noted, always fulfils (4.8). If f is less than 1, the union will threat to destroy $1-\gamma$ of the surplus. We find

$$\gamma^* = \min\left\{1, \frac{1-\delta_F}{\delta_F(1-\delta_F\delta_U)}\right\} = \min\left\{1, f(\delta_U, \delta_F)\right\}.$$

We are therefore led to investigate f's behavior. Fig. 4.1 shows that values of (δ_U, δ_F) actually exist for which $\gamma^* < 1$, implying that the model supports inefficient equilibria. Not only are the inefficient equilibria guaranteed to exist, but it is easily seen that, given δ_U, we must simply choose δ_F such that $f(\delta_U, \delta_F) < 1$, which is equivalent to[7]

$$1 > \delta_F > \tilde{\delta}_F(\delta_U) > \frac{1-\sqrt{1-\delta_U}}{\delta_U}. \tag{4.11}$$

Many of the following results assume that $\gamma^* < 1$, whose more precise meaning is the statement: "there exists $(\delta_U, \delta_F) \in (0,1)^2$ such that $\gamma^* < 1$", or, equivalently, if δ_U is fixed then it must be that $\delta_F \in (\tilde{\delta}_F, 1)$. First, the following result is considered:
Corollary 4.9. *If $\gamma^* < 1$, then $\Gamma_U(\delta_U, \delta_F)$ has an efficient perfect equilibrium with share vector $(\delta_F, 1-\delta_F)$.*

Proof. $\gamma^* < 1$ means that $\gamma^* = \frac{1-\delta_F}{\delta_F(1-\delta_F\delta_U)}$. Thus

$$b(\delta_U, \gamma^*\delta_F) = \frac{1-(1-\delta_F)/(1-\delta_F\delta_U)}{1-\delta_U(1-\delta_F)/(1-\delta_F\delta_U)} = \frac{1-\delta_U\delta_F - 1 + \delta_F}{1-\delta_U\delta_F - \delta_U + \delta_U\delta_F} = \delta_F. \tag{4.12}$$

Hence Proposition 4.8 shows us that the union's share is δ_F and, as the equilibrium is efficient, the sum must be 1 and the firm receives $1-\delta_F$. □

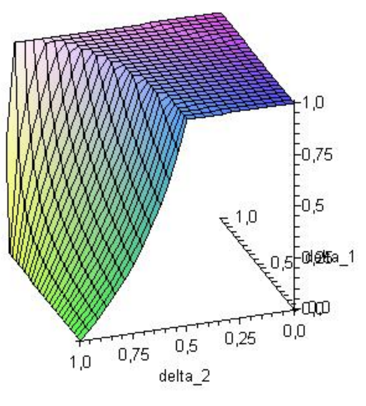

Figure 4.1: Plot of $\gamma^* = \min\big(1, f(\delta_U, \delta_F)\big)$. Pairs of discount factors exists such that surplus destruction is credible. Further, given δ_U, seeing what $\tilde{\delta}_F(\delta_U)$ should be is easy.

Note the result of the calculation in (4.12), as we shall use this in the following section.

[7] If we ignore the negative solution obtained from the quadratic equation.

We can now state the first of the two main results in this section. We wish to describe the possible outcomes of the bargaining in the model presented. The following theorem gives a lower bound for the union and the firm for any equilibrium.

Theorem 4.10. *For any equilibrium, the union's minimum payoff in* $\Gamma_U(\delta_U, \delta_F)$ *is* $b(\delta_U, \delta_F)$*. The firm's minimum payoff in* $\Gamma_U(\delta_U, \delta_F)$ *is* $1 - \delta_F$ *if* $\gamma^* < 1$ *and* $1 - b(\delta_U, \delta_F)$ *otherwise.*

The minimum payoff for one of the parties must provide information about the maximum possible payoff for the other party. Hence Theorem 4.10 shows that the best tactic for the union is described in Corollary 4.9, as the firm can do no worse than $1 - \delta_F$ in any equilibrium. Therefore, $1 - \gamma^*$ is referred to as the optimal destruction level (for the union). In contrast, the firm's best outcome is that of the original Rubinstein equilibrium from Proposition 4.8.

So far we have only focused on efficient equilibria, but we remedy this now. Theorem 4.11 is the cornerstone of the surplus destruction models and the last result presented in this section. It states that inefficient equilibria exists and characterizes these.

Theorem 4.11. *If* $\gamma^* < 1$*, any strategy yielding a payoff share* (ν_1, ν_2) *such that* $\nu_1 + \nu_2 \leq 1$ *that strictly dominates* $(b(\delta_U, \delta_F), 1 - \delta_F)$ *can be supported as an SPE in* $\Gamma_U(\delta_U, \delta_F)$*. If* $\gamma^* = 1$*, it must be that* $\nu_1 + \nu_2 = 1$ *and the payoff share is uniquely determined as* $(b(\delta_U, \delta_F), \delta_F b(\delta_U, \delta_F))$*: the Rubinstein share.*

Theorem 4.11 can be interpreted in the context of strikes. It states that, if no surplus is destroyed, there is one and only one outcome: the familiar Rubinstein equilibrium. Chapter 3 showed that the Rubinstein model has a unique equilibrium, and the nature of the Rubinstein model does not permit any surplus destruction. The statement of this theorem is stronger, however, as even though no surplus is destroyed it is a part of the model and it could be threatened.[8] Thus even in this model that potentially allows for surplus destruction, when it is not carried out we return to the original Rubinstein equilibrium.

If $\gamma^* < 1$, that is, the unions threat of surplus destruction is credible, Theorem 4.11 states that any outcome that strictly dominates the union's Rubinstein outcome is a possible SPE, and thus the union will always do better by committing itself to some level of surplus destruction (if this is credible).

The Manzini discussion. BSW proof that surplus destruction must be constant that is either γ^* or 1. Manzini [8] does not use this result, but investigates which conditions are necessary in order for the surplus destruction to be credible. First he shows that surplus destruction—he uses the term "harm" and "harming structure"—must occur in an infinite number of periods. The argument is simple. By way of contradiction let n be the last period in which any surplus is destroyed. As already noted, when surplus is no longer destroyed we return to the Rubinstein share. Thus, prior to performing the nth surplus destruction, the union is faced with the following choice: either not to engage in any destructive action, thereby getting the Rubinstein share of Π_{n-1}, or to perform the last surplus-destroying action and receive the Rubinstein share of $\Pi_n < \Pi_{n-1}$. This implies that any credible surplus destruction must occur an infinite number of times.

Manzini offers some additional necessary conditions on the surplus destruction

[8] The fact $\gamma^* = 1$ corresponds to the fact that any threat of surplus destruction is not credible.

structure primarily in Proposition 1. Most notable is his requirement[9] that, if $\gamma_0, \gamma_1 \ldots$ is an infinite sequence of surplus destruction factors, then it must fulfill

$$\gamma_n \leq \delta \sum_{i=0}^{\infty} \gamma_{n+i+1} \delta^{2i}, \tag{4.13}$$

where Manzini assumes that $\delta = \delta_U = \delta_F$. Does a constant sequence satisfy this requirement? We put $\gamma_i = \gamma$ for all i and obtain

$$\gamma \leq \delta\gamma \sum \delta^{2i} = \gamma \frac{\delta}{1-\delta^2},$$

where we have summed the geometric series. Thus for (4.13) to hold we must have

$$1 \leq \frac{\delta}{1-\delta^2} \iff 0 \leq \delta^2 + \delta - 1. \tag{4.14}$$

Solving this quadratic equation and omitting the negative solution, we obtain $\delta = \frac{-1+\sqrt{5}}{2} \approx 0.618$. This implies that for a constant sequence of surplus destruction a non-empty set of δ exists such destroying surplus is credible. It also shows a more intimate relation to the BSW model. For given δ_U, equation (4.11) gave us a lower bound for δ_F. What happens if we let $\delta_U = \delta_F$ in the BSW framework? We would then have to solve

$$\frac{1-\sqrt{1-\delta}}{\delta} = \delta \Rightarrow \delta(\underbrace{\delta^3 - 2\delta + 1}_{\delta=1 \text{ is root}}) = \delta(\delta-1)(\delta^2+\delta-1) = 0,$$

which we just solved in (4.14). Thus, if the firm and the union have the same discount factors, both Manzini and BSW agree on the necessary condition for the existence of an equilibrium with surplus destruction. Further, BSW proves that it is sufficient.

4.3.3 Limiting results and real time delay

As mentioned in the introduction, one of the main aims of extending the Rubinstein model should be to enable the real time delay that empirical studies motivate. As the time between periods decrease, that is $\Delta \to 0$, we move towards continuous time and despite always staying discrete, we describe the discount factors by $\delta_i = \exp(-r_i\Delta)$ and where $r_i > 0$ for $i = U, F$. With slightly imprecise notation, we write $\Gamma_i(\Delta) = \Gamma_i(\exp(-r_U\Delta), \exp(-r_F\Delta))$ for $i = U, F$. We shall denote the delay by $D(\Gamma_i(\Delta)) = T\Delta$, where T is the period in which we reach agreement. The maximum delay supported for a given (r_U, r_F) is denoted by $MD(r_U, r_F)$. When there is no risk of ambiguity, we refer to D and MD, respectively. Theorem 4.13 tells us, that given discount factors, we can always get arbitrarily close to the maximum delay.

We also investigate which equilibria are supported. When time between periods of negotiation shrinks, asking whether any given amount of surplus can be destroyed is still legitimate. Theorem 4.14 reveals the supported equilibria for both cases.

[9]Proposition 1, equation (1) in [8].

To begin our limiting analysis, we consider what happens with γ^* as $\Delta \to 0$. We first consider $f(\delta_U, \delta_F)$ and find that

$$\lim_{\Delta\to 0} f(\delta_U, \delta_F) = \lim_{\Delta\to 0} \frac{1-\exp(-r_F\Delta)}{\exp(-r_F\Delta)(1-\exp(-(r_U+r_F)\Delta))} = \frac{r_F}{r_U+r_F} < 1$$

where we have used l'Hôpital's rule. Hence when Δ approaches 0, f is strictly less than 1 from a certain Δ_0 and we get

$$\gamma^* = f(\delta_U, \delta_F) < 1 \text{ for } \Delta \in (0, \Delta_0) \text{ and } \gamma^* \to \frac{r_F}{r_U+r_F} \text{ as } \Delta \to 0. \tag{4.15}$$

This in itself is already an important result as, when combined with Theorem 4.11, it shows that, as the time between periods decreases, the existence of inefficient equilibria is guaranteed. The payoff share for the union in these inefficient equilibria is given by

$$\lim_{\Delta\to 0} b(\delta_U, \delta_F) = \lim_{\Delta\to 0} \frac{1-\exp -r_F\Delta}{1-\exp -(r_U+r_F)\Delta} = \frac{r_F}{r_U+r_F}$$

using l'Hôpital's rule once more.

Lemma 4.12. *For a feasible payoff vector (x, y) where $x+y \leq \Pi$, there exists a small enough Δ such that (x, y) is an SPE payoff of $\Gamma_U(\Delta)$ if and only if $x > r_F\Pi/(r_U+r_F)$ and $y > 0$.*

Theorem 4.14 generalizes this result, but for now, we return to investigate the maximum delay supported. If $x + y < \Pi$, we must have delayed agreement either due to discounting or surplus destruction. For any T, let α be the remaining surplus to be split. We then have

$$x\exp((T-1)r_U\Delta) + y\exp((T-1)r_F\Delta) = \alpha \leq \Pi$$

or, equivalently,

$$x\exp(r_U(D-\Delta)) + y\exp(r_F(D-\Delta)) = \alpha \leq \Pi. \tag{4.16}$$

We want to maximize D in (4.16) as a function of x, y and α. For any given α, if either x or y increases then D would have to decrease. Analogously, for a given x and y, if α increases then D can increase as well. We must therefore obtain an upper bound of the delay if we pick the largest α and the smallest pair of x and y. That is, $\alpha = \Pi$, $x = r_F/(r_U + r_F)$ and $y = 0$. We do not know whether we can realize this upper bound or even get close, but we are certain that we cannot get above it. Inserting the chosen level of constants in (4.16), letting $\Delta \to 0$ and denoting D by MD, we find

$$\lim_{\Delta\to 0} \frac{r_F}{r_U+r_F}\exp(r_U(MD-\Delta)) = \Pi \quad \Longrightarrow \quad MD = \frac{1}{r_U}\log\left(\frac{r_U+r_F}{r_F}\Pi\right). \tag{4.17}$$

We refer to MD as the maximum delay, and the following theorem states that, although MD cannot be reached, we can get arbitrarily close. Before the theorem is stated note that for MD to be meaningful it must be a strictly positive number. This is only guaranteed if $\Pi > r_F/(r_U + r_F)$, which is a point that BSW—in our view—miss as they normalize Π.

Theorem 4.13. *For any $0 < \epsilon < MD(r_U, r_F)$ with MD defined as in equation (4.17), we can get a delay $D > MD(r_U, r_F) - \epsilon$ in agreement in the game $\Gamma_U(\Delta)$ if Δ is small enough. That is,*

$$\forall 0 < \epsilon < MD(r_U, r_F)\ \exists \Delta > 0 : D > MD - \epsilon.$$

We now return to the question of which feasible payoff vectors are supported as an SPE. This generalizes Lemma 4.12. Our startingpoint is the idea that, as the time between negotiation periods decreases, destroying any given amount of surplus might not be reasonable. Some kinds of surplus are easily destroyed, such as computer data. Other kinds of surplus are more difficult to destroy, such as customer relationships built up over many, many years. Case 3 provides empirical motive for the limitation on surplus destruction.

These considerations are formalized as follows. Instead of allowing γ to vary freely between 0 and 1, we introduce a function $g : [0, \infty] \to [0, 1)$ such that $\gamma \in [g(\Delta), 1]$. We assume that $\lim_{\Delta \to 0} g(\Delta) = 1$ for obvious reasons. Further, we assume that g is differentiable with $g'(0) \leq 0$. The latter condition simply states that, as Δ tends to 0, g will tend to 1 from below.[10] The union's minimum payoff is unchanged by Proposition 4.8, as this is the Rubinstein equilibrium occurring when $\gamma = 1$, which will always be a possible scenario. The maximum payoff for the union will decrease, however. The union would still prefer to choose as low a γ as possible, but our original requirement of a credible threat given by (4.8) must still hold. On the other hand they cannot pick a γ smaller than $g(\Delta)$ and hence they will have to pick the largest: that is

$$\gamma^* = \max \left\{ g(\Delta), \min \left\{ 1, \frac{1 - \exp(-r_F \Delta)}{\exp(-r_F \Delta)(1 - \exp(-(r_U + r_F)\Delta))} \right\} \right\}. \tag{4.18}$$

Fortunately, (4.18) simplifies in the limit. For $\Delta \to 0$, we know that $g(\Delta) \to 1$, whereas an Δ_0 exists such that

$$\frac{1 - \exp(-r_F \Delta)}{\exp(-r_F \Delta)(1 - \exp(-(r_U + r_F)\Delta))} \to \frac{r_F}{r_U + r_F} < 1 \text{ for all } \Delta \in [\Delta_0, 1]$$

and hence $\gamma^* = g(\Delta)$ for $\Delta \in [\Delta_0, 1]$. According to Corollary 4.9, the union's highest payoff share is $b(\delta_U, \gamma^* \delta_F)$, which in the limit becomes $b(\delta_U, g(\Delta)\delta_F)$. We therefore have

$$\begin{aligned}
\lim_{\Delta \to 0} b(\delta_U, g(\Delta)\delta_F) &= \lim_{\Delta \to 0} \frac{1 - g(\Delta)\exp(-r_F \Delta)}{1 - g(\Delta)\exp(-(r_U + r_F)\Delta)} \\
&= \frac{r_F - g'(0)}{r_U + r_F - g'(0)} > \frac{r_F}{r_U + r_F},
\end{aligned}$$

once again using l'Hôpital's rule. The firm's lowest payoff share will therefore be

$$\frac{r_U}{r_U + r_F - g'(0)}.$$

We are now ready to generalize Lemma 4.12.

[10] First, note that, strictly speaking $g'(0)$ is undefined as g is mapping from a closed set. We would have to map from $(-\epsilon, \infty)$ for any $\epsilon > 0$. This, however, is a technical mathematical detail that does not concern us. Further, the assumption that g is differentiable or even continuous is not important but rather is the limiting properties of g. Our presentation is based on convenience.

Theorem 4.14. *If $\gamma \geq g(\Delta)$ then for any feasible payoff vector (x, y) where $x + y \leq \Pi$, there exists a small enough Δ such that (x, y) is an SPE payoff of $\Gamma_U(\Delta)$ if and only if $x > r_F/(r_U + r_F)$ and $y > r_U\Pi/(r_U + r_F - g'(0))$. The model has a unique SPE if and only if $g'(0) = 0$.*

At first it might seem peculiar that $g'(0) = 0$ guarantees a unique equilibrium, but consider the Taylor expansion of g around $g(0) = 1$

$$g(\Delta) = 1 + g'(0)\Delta + \mathcal{O}(\Delta^2) \tag{4.19}$$

Now, if $g'(0) = 0$, then $g(\Delta)$ will approach 1 much faster, at $\mathcal{O}(\Delta^2)$, than Δ itself and the only equilibrium that survives in the limit is the Rubinstein equilibrium with no surplus destruction at all. If, in contrast, $g'(0) = -\infty$, that is, $g'(0)$ is a very negative number, then introducing the additional restriction through g in our model does not affect the limiting set of equilibrium payoffs. The following case motivates the introduction of the g-function as an upper bound empirically.

Case 3: strike by the Danish Nurses' Organization, 2008
This case shows that legislation or exogenous limits may exists, that the BSW model describes by g. We continue to investigate the strike by the Danish Nurses' Organization described in Case 1.

We have already criticized the all-or-nothing approach by FG. Besides that it might not be the optimal strategy for these nurses. Legislation in Denmark also imposes constraints on their strikes. Due to the low wages, (public) hospitals have had difficulty in attracting nurses. Thus, when the strike began, many hospital departments were already experiencing a shortage of nurses. Because of the nature of the health sector, legislation requires a minimum number nurses at work. The initial shortage had already reduced the number of many departments below this legal requirement, and thus they could not strike. This greatly decreased the flexibility and even the possibility of surplus destruction—that is a non-trivial g—by the Danish Nurses' Organization during the strike and strongly influenced the effectiveness of the strike.

4.3.4 "Non-uniform" folk theorem

In this section we prove that almost any feasible payoff vector can be realized in an SPE if we make the discount factors large enough. We first choose δ_U and then δ_F as a function of δ_U, which is why the folk theorem is called "non-uniform". First, recall from (4.11) that $\tilde{\delta_F}(\delta_U)$ gives a lower bound for δ_F given δ_U such that $\gamma^* < 1$. Further, we find that

$$\frac{b(\delta_U, \delta_F)}{\delta_U} = \frac{(1 - \delta_F)\delta_F}{(1 - \delta_U\delta_F)^2} > 0 \tag{4.20}$$

and

$$\frac{b(\delta_U, \delta_F)}{\delta_F} = \frac{-(1 - \delta_U\delta_F) + (1 - \delta_F)\delta_U}{(1 - \delta_U\delta_F)^2} = \frac{\delta_U - 1}{(1 - \delta_U\delta_F)^2} < 0, \tag{4.21}$$

which shows that the union's payoff is increasing in its own discount factor and decreasing in the firm's discount factor. As we first choose δ_U, we can always choose δ_F large enough to render the union's payoff, that is $b(\delta_U, \delta_F)$, arbitrarily small.

Theorem 4.15 (folk theorem). *For any individual rational*[11] *vector* (x, y) *such that* $x + y \leq \Pi$ *and* $x, y > 0$ *there exists a fixed* δ_U^0 *such that* (x, y) *is an SPE for* $\delta_U \in (\delta_U^0, 1)$ *and* $\delta_F \in (\bar{\delta}_F(\delta_U^0), 1)$.

4.3.5 Conclusion on the permanent surplus destruction model

Introducing the possibility of surplus destruction—whether actually carried out or only threatened—gives the union a higher payoff. This follows from the fact that the firm becomes indifferent between accepting a worse offer or sharing a smaller surplus. A mathematical criterion for when a threat is credible has been presented and the union's optimal level of surplus destruction γ^* defined. Such a level depends on the discount factors of the firm and union, and it is not guaranteed that surplus will be destroyed at all. That is, the optimal level of surplus destruction might be $\gamma = 1$. On the other hand, for a fixed δ_U, (4.11) gives a lower bound for δ_F such that surplus destruction is credible. This lower bound is also used in the folk theorem BSW present, which states that almost any feasible payoff vector is attainable in the BSW model if the discount factors are determined as described.

For the limiting situation in which the time between periods approaches 0, the model shows that the optimal level of surplus destruction will induce (a threat of) real surplus destruction. Further, the model supports a real time delay even as bargaining friction disappears: that is, time between periods approaches 0. An upper bound for this delay was found and described in Theorem 4.13. Further, it shows that one can get arbitrarily close to this upper bound if the time between offering periods decreases sufficiently.

As the time between periods decreases, destroying any given amount of surplus might not be credible. We have modelled this and found that, although it restricts the set of equilibria, it still permits the existence of multiple (inefficient) equilibria as the time between offering periods decreases sufficiently. The real-life Cases 1 and 3 motivated this feature of the model.

Theorem 4.13 describes the maximum delay attainable in the model, whereas Theorem 4.14 describes the levels of surplus destruction supported. The folk theorem quantitatively describes the supported equilibria—efficient as well as inefficient—but the delay and the level of surplus destruction cannot be chosen independently. Given a supported equilibrium, the degree of surplus destruction is negatively correlated with the length of delay. This means that, for a given inefficient equilibrium, this inefficiency must come from a combination of delay and surplus destruction, but we cannot freely apply Theorem 4.13 or Theorem 4.14. If one likes a high level of delay, one must accept that the supported surplus destruction will decrease and vice versa.

In [6] BSW raise the question of what would happen if the firm has the power to destroy surplus as well. It turns out that all the equilibria found in the preceding analysis continue to exist, while new ones are added. The spectrum of possible equilibria simply grows and, for instance, the union's worst payoff becomes less than the Rubinstein payoff. In this presentation we have, just as BSW, chosen to focus on the model in which only one player destroy surplus to simplify and clarify the analysis and the tools used. BSW further motivates this choice, claiming that it aids in emphasizing the features of the model.

[11] A payoff that is at least the minimax payoff of the players. See Osborne & Rubinstein [11], p. 143.

Continuing this line of thought, whereas AZ assume that the union only has destructive powers in periods following rejection of their proposal, BSW do not make this simplification. Rather, this assumption turns out to be a result of BSW's model and is shown to be justified. It includes the possibility, but as Proposition 4.8 shows it will never be profitable for the union to destroy surplus after rejecting the firm's offer.

We believe that the BSW model is a good example of scientific research. Not only do they generalize the model investigated by AZ, but the higher level of abstraction presented in the BSW model enables the assumptions made by AZ to be justified. Further, the model is well motivated, and being able to destroy any fraction of the surplus certainly makes sense. Some trivial generalizations of their model including new results are mentioned but left out to improve transparency. Further, we have been able to show that the necessary conditions found by BSW and Manzini are equivalent in the case where firm and union have the same discount factors.

Part II

Our analysis

CHAPTER 5

Our model

5.1 Introduction

In Chapter 4 we analyzed and discussed the models of Fernandez & Glazer and Busch, Shi & Wen. We pointed out the strengths and weaknesses of both models. Our main critique of the FG model is the harsh division: either all the surplus is kept intact or it is destroyed completely. However, their idea of a reappearing surplus stream seems fruitful to us, even though earlier strikes having no effect on subsequent surplus is a critical assumption. BSW, in contrast, took care of the impact of strikes on surplus, but their surplus destruction is permanent. Thus, the destroyed surplus cannot be regained. This is surely due to the structure of their model: they do not have reappearing surplus but a single surplus about which the parties bargain. In our model a compromise is possible, whereas the FG and BSW models are still included as special cases.

In this chapter we aim to remedy some of the shortcomings and to combine the strengths of both models in an even better model. Empirical evidence suggests that a reappearing surplus is a precise way to describe the model in reality. Many strikes—especially long-term strikes such as the DB strike—are called on and off over an extended period of time, and the FG framework captures this beautifully. Whereas FG take the reappearing surplus to be a constant, we hypothesize that it depends on several factors, as we believe that some kind of surplus destruction in one period might reduce the surplus that reappears in the following period. Customers might leave for a competitor or find alternative ways to satisfy their needs for the service offered by the firm.[1] This feature of non-constant reappearing surplus is also fruitful for strikes and surplus destruction without breaks.

To find a better balance the transition equation is introduced. It describes how the surplus reappears between periods and we will analyze its asymptotic behavior as well as its rate of convergence.

[1] For instance, people in Berlin might commute by cycling thus cutting DB's revenue.

Another novel feature of our model is the introduction of surplus regeneration. This allows the firm to regenerate some of the destroyed surplus and thereby reduce the damage inflicted by the union through its destructive actions. Below we prove that SPE involving surplus regeneration exist, which serves as our motive for introducing it in our model. The regeneration only directly affects the firm—which is formally seen through the utility functions—and can therefore be thought of as an opting-out option. This is not a traditional opting-out option though. Usually, opting-out defines what the parties will receive when no agreement can be reached. In this case the opting out option is purely an alternative for the firm not involving the union.

Specifically, if the union has a number of workers on strike the firm might be able to hire some freelance workers and thereby regenerate some of the surplus destroyed by the union. However, regenerating the entire surplus might be impossible or at least unprofitable. There might be a shortage of freelance workers or the cost of employing them might be higher than the expected outcome.

5.2 The model

We assume that the firm has an initial surplus of $\Pi_0 > 0$ about which the parties can bargain. The existing wage contract between the union and the firm is denoted $w_0 = (x_0, y_0)$, where $x_0 \in [0,1]$ is the union's share, $y_0 \in [0,1]$ is the firm's share and $x_0 + y_0 \leq 1$.[2] In odd periods, the union makes a wage offer $w_t = (x_t, y_t) = (x_t, 1-x_t)$ and in even periods the firm proposes a wage contract w_t. Analogously, in odd periods the firm has to decide whether or not to accept the proposed w_t, and in even periods the union has to decide. If the firm accepts the wage contract w_1 in period 1, the game ends and the union receives $x_1\Pi_0$ and the firm $(1 - x_1)\Pi_0$. If the firm rejects, the union decides a level of surplus destruction, which we denote $1-\gamma_t$, where $\gamma_t \in [0,1]$ and $t \in \mathbb{N}$. Whenever $\gamma_t = 1$, we say that there is no surplus destruction in period t. For $\gamma_t = 0$, all surplus is destroyed.

We introduce the option that the firm can regenerate surplus. After the union decided upon γ_t, the firm now has the option of regenerating some of the destroyed surplus. We denote the degree of surplus regeneration as $\alpha_t \in [0,1]$. There are some restrictions on α_t that we will discuss further. For now we solely consider the possibility of surplus regeneration. After the firm has made its choice of α_t, the surplus of period 1 is divided between the union and the firm according to the pre-existing contract: that is, the union receives $x_0\gamma_1\Pi_0$ and the firm receives $y_0\gamma_1\Pi_0$, but in addition the firm receives its opting-out share: that is, $\alpha_1\Pi_0$. Thus, the firm receives $y_0\gamma_1\Pi_0 + \alpha_1\Pi_0$ in total. The game continues to period 2, but first we describe the second new feature of our model: the transition equation. For $\tau \in [0,1]$ we define

$$\Pi_t := (\gamma_t + \alpha_t)\Pi_{t-1}^{\tau}\Pi_0^{1-\tau}. \tag{5.1}$$

Note that Π_t is the surplus to be divided at the end of period t.[3] τ balances reappearing surplus with permanent surplus. Note that if $\gamma_t = \gamma$ and $\alpha_t = \alpha$ for all t, we get

$$\Pi_t = (\gamma + \alpha)^{\sum_{i=0}^{t-1}\tau^i}\Pi_0,$$

[2] Including the case $x_0 = 0$ and $y_0 = 0$ is important even though this seems odd. This is explained below.

[3] This implies that the term $\Pi_{t-1}^{\tau}\Pi_0^{1-\tau}$ is the surplus at the beginning of period t.

which implies that Π_t only depends on the initial surplus and not (directly) on the surplus of the preceding period.

Consider period 2. The surplus available is $\Pi_2 = (\gamma_1 + \alpha_1)\Pi_0$. Now the firm makes the offer w_2 and the union has to reply. If the union accepts, the payoff will be divided according to w_2. If it rejects, the union and the firm may decide upon γ_2 and α_2. Afterwards, the union and the firm divide the payoff of period 2 according to the old wage contract and the game continues to period 3, which is similar to period 1, in which the surplus given by (5.1) is now $\Pi_3 = (\gamma_2 + \alpha_2)\Pi_2$. Fig. 5.1 presents the first two periods of the game.

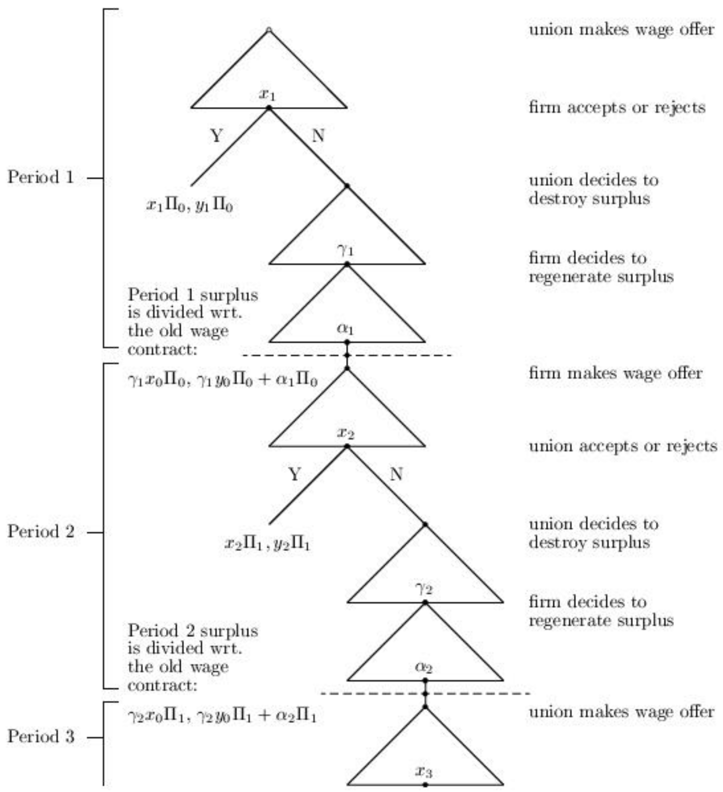

Figure 5.1: First two periods of the extensive game tree of the model

We now investigate the feasible levels of surplus regeneration. Firstly, it is never possible to regenerate more surplus than has been destroyed: that is, $\alpha_t \in [0, 1-\gamma_t]$.[4] Secondly, surplus has a price and this cost is given by a fraction of the total available surplus described by $c(\alpha_t) \in \mathbb{R}_+$. Assume that c is a strictly convex function with $c(0) = 0$ and that a unique $\hat{\alpha} \in (0, 1]$ exists such that $c(\hat{\alpha}) = \hat{\alpha}$. Note that this implies that $c(\alpha) > \alpha$ for any $\alpha > \hat{\alpha}$, and thus $\hat{\alpha}$ is referred to as the maximal level of non-negative surplus regeneration. This implies that opting out will only be profitable and possible for

$$\alpha_t \in [0, 1 - \gamma_t] \cap [0, \hat{\alpha}]. \tag{5.2}$$

Further, $\hat{\alpha}$ can be seen as a function of (the function) c. Lastly, $\hat{\alpha} = 0$ is a pathological case in which it can never be profitable to opt out. This removes the desired

[4] Imagine for example, the DB strike. Hiring more engine drivers than are on strike does not generate additional surplus due to the restricted numbers of trains operating.

feature in our model, and any novel results will have $\hat{\alpha}(c) > 0$. Fig. 5.2 shows a generic example of a cost function as we consider it in the remainder of the chapter.

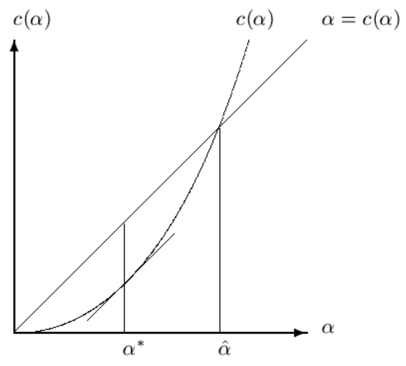

Figure 5.2: Plot of $c(\alpha)$

The distance between α and the cost function $c(\alpha)$ can be seen as the net surplus arising from surplus regeneration. Since c is a convex function, we can find a maximum for $\alpha - c(\alpha)$, which we denote α^*. Due to the properties of c, we must have that $\alpha^* \in (0, \hat{\alpha})$.

To conclude the description of our model, we define the utility function of the union and firm to be

$$\text{union: } \sum_{t=1}^{\infty} \delta_U^{t-1} \gamma_t \Pi_{t-1} x_t, \quad \text{and firm: } \sum_{t=1}^{\infty} \delta_F^{t-1} \left(\gamma_t \Pi_{t-1} y_t + (\alpha_t - c(\alpha)) \Pi_{t-1} \right). \quad (5.3)$$

Note that the union does not benefit from any surplus regenerated and that the firm does not need to share any of the regenerated surplus. Further, if agreement is reached in period T the game terminates and any future level of surplus destruction or therefore is therefore left undefined. They are still part of the utility function, however, and hence we put $\gamma_t = 1$ and $\alpha_t = 0$ for all $t \geq T$.

5.3 Past models as special cases

In this section we show how all three models in focus earlier in this thesis can be seen as special cases of our more general model. This follows by choosing the involved parameters in a clever way and showing that both the structure and the utility function essentially reduce to what is described in the past models. Further, as part of the analysis of BSW, we show Lemma 5.1, which states that an SPE exists involving surplus regeneration and thereby justifies the introduction of this feature.

5.3.1 The Fernandez & Glazer model

The FG model is a special case of our model described above. First, we put $\tau = 0$. FG use the all-or-nothing approach in their model, and thus we set $\gamma_t \in \{0, 1\}$ and notice once again that $\gamma_t = 1$ in period t corresponds to no strike when there is no agreement and $\gamma_t = 0$ corresponds to strike in case of disagreement. Moreover, we choose c in a way such that $\hat{\alpha} = 0$. This ensures that surplus regeneration is not feasible, and thereby $\alpha_t = 0$ for all t. We see from our transition equation (5.1) that it reduces to

$$\Pi_t = (\gamma_t + \alpha_t)\Pi_0 = \gamma_t \Pi_0. \tag{5.4}$$

This corresponds to the reappearing surplus as stated in Section 4.2. Referring to $w_0 = (x_0, y_0) = (x_0, 1 - x_0)$ as the old wage contract, we now define the wage contracts for all periods to be

$$x_t = \begin{cases} x_0, & \text{if } t < T \\ x_T, & \text{if } t \geq T \end{cases} \quad \text{and} \quad y_t = \begin{cases} y_0 = 1 - x_0, & \text{if } t < T \\ y_T = 1 - x_T, & \text{if } t \geq T \end{cases} \tag{5.5}$$

where T is the period when agreement is reached and x_T is the new wage contract agreed on in period T. If $\gamma_t = 0$, (5.4) shows that $\Pi_t = 0$ in this period and therefore no surplus is available to either the union or the firm. Analogously, $\Pi_t = \Pi_0$ when there is no strike. The utility functions (5.3) thus become

$$\text{union: } \sum_{t=1}^{\infty} \delta_U^{t-1} \gamma_t \Pi_0 x_t \quad \text{and firm: } \sum_{t=1}^{\infty} \delta_F^{t-1} \gamma_t \Pi_0 (1 - x_t), \tag{5.6}$$

where we recall that $\gamma_t = 1$ for all $t \geq T$.

We turn our attention to finding SPE where the firm can regenerate surplus to motivate the introduction of surplus regeneration. First, we assume that $\hat{\alpha} > 0$ so that we allow for $\alpha > 0$. Based on the range of possible α's (5.2), regenerating $\alpha_t = \alpha^*$ is always profitable for the firm when the union strikes, that is $\gamma_t = 0$, and that the firm performs no surplus regeneration, $\alpha_t = 0$, when there is no strike, $\gamma_t = 1$. Surplus regeneration does not positively affect the utility of the union: that is, it can only benefit the firm. Thus, surplus regeneration, is accounted for only in the utility function of the firm. We can write (5.6) as

$$\text{union: } \sum_{t=1}^{\infty} \delta_U^{t-1} \gamma_t \Pi_0 x_t \quad \text{and firm: } \sum_{t=1}^{\infty} \delta_F^{t-1} \left(\gamma_t \Pi_0 (1 - x_t) + (\alpha_t - c(\alpha_t))\Pi_0 \right),$$

Following up on the discussion on efficient equilibria in Section 4.2.2 the possibility of surplus regeneration does not affect the equilibrium strategy of the union. However, the firm's equilibrium strategy now includes surplus regeneration. We do not reproduce all findings from Section 4.2.2, as this is trivial. It is representative for the whole discussion in this section to consider only Lemma 4.6, and we concentrate on this.

Define $w^* = (x^*, y^*) = (x^*, 1 - y^*)$. Consider the following strategy of the union,

$$s_U^* = \begin{cases} w_t = w^*, & \text{when } t = 1, 3, 5, \ldots \\ \gamma_t, & \begin{cases} 0, & \text{when } t = 1, 3, 5, \ldots \text{ and the firm has rejected } w_t = w^* \\ 1, & \text{for all } t = 2, 4, 6, \ldots \end{cases} \\ Y, & \text{when } t = 2, 4, 6, \ldots \text{ and the firm's offer is } x_t \geq x^* \\ N, & \text{when } t = 2, 4, 6, \ldots \text{ and the firm's offer is } x_t < x^* \end{cases}$$

and of the firm

$$s_F^* = \begin{cases} w_t = w^*, & \text{when } t = 2,4,6,\ldots \\ \alpha_t, & \begin{cases} 0, & \text{when the union plays } \gamma_t = 1 \\ \alpha^*, & \text{when the union plays } \gamma_t = 0 \end{cases} \\ Y, & \text{when } t = 1,3,5,\ldots \text{ and the union's offer is } x_t \leq x^* \\ N, & \text{when } t = 1,3,5,\ldots \text{ and the union's offer is } x_t > x^*. \end{cases}$$

Again, we add the deviation property to this strategy: that is, if the union deviates from playing s_U^*, both players are forced to play the equilibrium equivalent to Lemma 4.3: that is, in our case, the equilibrium strategies that support x_0 and y_0 in (5.5).

Based on these strategies, we can now justify that SPE exist with surplus regeneration. Recall equations (4.5) and (4.6). Since surplus regeneration does not directly affect the utility of the union, the union will stick to following strategy s_U^* in equilibria for $w_0 \leq w^* \leq w'$.[5] Intuitively, the introduction of surplus regeneration does not weaken the bargaining power of the union about the original surplus. Surplus regeneration is only important for the firm. The firm will regenerate surplus whenever the union decides to strike, but introducing surplus regeneration does not affect the possible equilibria that can be achieved. The only effect of surplus regeneration in the FG model is that strikes do hurt the firm less during periods of strikes. However, they do not affect the outcome of the wage negotiations.

The only thing left is to check for the Bolt addition. Similar to the Bolt discussion in Section 4.2.2 we have to analyze the condition under which the maximum contract w' can be achieved. Again, we are using the non-concession strategy by the firm as an alternative strategy. If the firm is obliged to play the non-concession strategy, its payoff equals $(1-\delta_F)\sum_{t=1}^{\infty}\delta_F^{t-1}(\gamma_t\Pi_0 y_t + (\alpha_t - c(\alpha_t))\Pi_0)$. Since the union's strategy tells it to strike in every odd-numbered period after the firm has rejected its offer (which it does according to the no-concession strategy) $\gamma_t = 0$ for all odd t. However, now the union has the opportunity to regenerate surplus, which it will do. Thus, $\alpha_t = \alpha^*$ for odd t. Define $\alpha_c^* = \alpha^* - c(\alpha^*)$. The no-concession payoff can thereby be stated as

$$\begin{aligned}(1-\delta_F)\sum_{t=1}^{\infty}\delta_F^{t-1}(\gamma_t\Pi_0 y_t + (\alpha_t - c(\alpha_t))\Pi_0) &= (1-\delta_F)\big[(\delta_F + \delta_F^3 + \ldots)\Pi_0 y_0 \\ &\quad + (1 + \delta_F^2 + \delta_F^4 + + \ldots)\alpha_c^*\Pi_0\big] \\ &= (1-\delta_F)\big[\frac{\delta_F}{1-\delta_F^2}\Pi_0 y_0 + \frac{1}{1-\delta_F^2}\alpha_c^*\Pi_0\big] \\ &= \frac{1}{1+\delta_F}\Pi_0(\delta_F y_0 + \alpha_c^*).\end{aligned}$$

We can now analyze the cases in which the firm prefers the no-concession outcome to the maximum wage contract. This is when

$$\frac{1}{1+\delta_F}\Pi_0(\delta_F y_0 + \alpha_c^*) > \delta_F\frac{1-\delta_U}{1-\delta_U\delta_F}\Pi_0 y_0.$$[6]

This inequality can be reduced to

$$\frac{\alpha_c^*}{\delta_F y_0}(1-\delta_F\delta_U) > \delta_F - \delta_U. \quad (5.7)$$

[5]Where w' is defined in Section 4.2.2.

[6]Here, the right and side equals the maximum contract outcome for the firm.

Remember that, without surplus regeneration, the firm preferred the maximum contract whenever $\delta_F \geq \delta_U$. The left-hand side of inequality (5.7) is strictly greater than zero (remember that $\delta_F, \delta_U \in (0,1)$). It therefore follows that the cases with no surplus regeneration continue to exist: that is, whenever $\delta_U > \delta_F$ the firm prefers the no-concession strategy to achieving the maximum wage contract. However, since the left-hand side is strictly greater than zero, there are cases where $\delta_F > \delta_U$ and the firm still prefers the no-concession result depending on the parameters α_c^* on y_0. Especially for the cases where $\alpha_c^* = y_0$, inequality (5.7) reduces to $\frac{1}{\delta_F}(1-\delta_F\delta_U) > \delta_F - \delta_U$ which can be further reduced to yield $1 > \delta_F^2$. This means that for regardless of the discount factors of the union or the firm, the firm simply prefers the no-concession result to the maximum wage contract if $\alpha_c^* = y_0$. This case should clarify the fact that, with the introduction of surplus regeneration, the bargaining situation has changed and the maximum contract is sometimes not subject to an SPE at all. It does not help the understanding of the model to give a complete analysis of inequality (5.7) which is why we omit it here. The result to keep in mind is that introducing surplus regeneration indeed changes the range of possible contracts even for efficient equilibria.

We can conclude about efficient equilibria that, with surplus regeneration, Lemma 4.6 continues to hold with the above strategies s_U^* and s_F^* and their deviation property if δ_U and δ_F satisfy inequality (5.7). We examine now the case for inefficient equilibria as stated in Section 4.2.3 with surplus regeneration.

As shown above, inefficient equilibria are achieved through a continuous period of strikes before reaching a final agreement. In order to achieve this, equilibrium strategies must include a deviation property. In the above case, deviation by the union is penalized by playing the equilibrium of Lemma 4.3, while deviation by the firm is penalized by playing the equilibrium of Lemma 4.4, which is equivalent to the Rubinstein equilibrium when sharing a one-time profit of Π_0. Again, we have to state the constraints on possible wage contracts to be generated in an inefficient equilibrium.

If $\tilde{w}$ is the contract to be bargained about, it must hold as stated in Section 4.2.3 that the union prefers striking for T periods followed by an agreement of $\tilde{w}$ to agreeing on w_0 immediately in the first round: that is, $\delta_U^T \hat{w} \geq w_0$. The more interesting result is the situation of the firm. In the T periods of the strike, that is, the union chooses $\gamma_t = 0$ for $t \leq T$, the firm does now have the possibility of opting-out. The above shows that, if $\gamma_t = 0$, the firm's best response is to play $\alpha_t = \alpha^*$. Thus, it receives $\alpha^*\Pi_0$ in every period when the union strikes.[7] As we assume a strike in each of the T pre-agreement periods, we define the present value of the accumulated surplus regeneration to be $\psi := \sum_{t=1}^{T} \delta_F^{t-1}\alpha^*\Pi_0$. This is a finite geometric series that can be rewritten as $\psi = \frac{\delta_F^T - 1}{\delta_F - 1}\alpha^*\Pi_0$. Thus, for $\tilde{w}$ to be an equilibrium contract, $\Pi_0 - \bar{z} \leq \delta_F^{T-1}(\Pi_0 - \hat{w}) + \psi$ must apply. This can be rearranged as

$$\tilde{w} \leq (1 - \delta_F^{1-T})\Pi_0 + \delta_F^{1-T}\bar{z} + \delta_F^{1-T}\psi.$$

It follows that

$$\delta_U^{-T}\hat{w} \leq \tilde{w} \leq (1 - \delta_F^{1-T})\Pi_0 + \delta_F^{1-T}\bar{z} + \delta_F^{1-T}\psi. \tag{5.8}$$

[7] Note that inefficient equilibria are also achieved by having periods of strikes mixed with periods of no strikes before reaching agreement. The results are derived similar to those here, including the payoffs in the peaceful periods to the present value computation of $\tilde{w}$.

Therefore with surplus regeneration, Theorem 4.7 holds with inequality (5.8) instead of (4.7). The strategies that support these inefficient equilibria are basically the same as the ones stated in Section 4.2.3. The only difference is that the firm is now allowed to perform surplus regeneration, whenever the union decides to strike. For convenience we rewrite the union's and the firm's strategy, respectively.

$$\hat{s}_u = \begin{cases} x_t & \begin{cases} > (1-\delta_f^{1-T})\Pi + \delta_f^{1-T}\bar{z}, & \text{if } t \in \{1,\dots,T\} \\ = \hat{w}, & \text{when } t = T+1, T+3, \dots \text{ and } T+1 \text{ is odd} \\ = \hat{w}, & \text{when } t = T+2, T+4, \dots \text{ and } T+1 \text{ is even} \end{cases} \\ \\ \gamma_t, & \begin{cases} 0, & \text{when } t \in \{1,\dots,T\} \\ & \text{when } t = T+1, T+3, \dots,\ T+1 \text{ even, and the firm has rejected } x_t = \hat{w} \\ & \text{when } t = T+2, T+4, \dots,\ T+1 \text{ odd, and the firm has rejected } x_t = \hat{w} \\ 1, & \text{for all } t = T+2, T+4, \dots,\ T+1 \text{ odd} \\ & \text{for all } t = T+1, T+3, \dots,\ T+1 \text{ even} \end{cases} \\ \\ Y, & \text{when } t = T+2, T+4, \dots, \text{ for } T+1 \text{ odd and } y_t \geq \hat{w} \\ & \text{when } t = T+1, T+3, \dots, \text{ for } T+1 \text{ even and } y_t \geq \hat{w} \\ \\ N, & \text{when } t \in \{1,\dots,T\} \\ & \text{when } t = T+2, T+4, \dots, \text{ for } T+1 \text{ odd and } y_t < \hat{w} \\ & \text{when } t = T+1, T+3, \dots, \text{ for } T+1 \text{ even and } y_t < \hat{w} \end{cases}$$

$$\hat{s}_f = \begin{cases} y_t & \begin{cases} = w_0, & \text{if } t \in \{1,\dots,T\} \\ = \hat{w}, & \text{when } t = T+2, T+4, \dots \text{ and } T+1 \text{ is odd} \\ = \hat{w}, & \text{when } t = T+1, T+3, \dots \text{ and } T+1 \text{ is even} \end{cases} \\ \\ \alpha_t, & \begin{cases} 0, & \text{when the union plays } \gamma_t = 1 \\ \alpha^*, & \text{when the union plays } \gamma_t = 0 \end{cases} \\ \\ Y, & \text{when } t = T+1, T+3, \dots, \text{ for } T+1 \text{ odd and } x_t \leq \hat{w} \\ & \text{when } t = T+2, T+4, \dots, \text{ for } T+1 \text{ even and } x_t \leq \hat{w} \\ \\ N, & \text{when } t \in \{1,\dots,T\} \\ & \text{when } t = T+1, T+3, \dots, \text{ for } T+1 \text{ odd and } x_t > \hat{w} \\ & \text{when } t = T+2, T+4, \dots, \text{ for } T+1 \text{ even and } x_t > \hat{w} \end{cases}$$

What is the effect of the introducing of surplus regeneration. A comparison of inequality (4.7) and (5.8) shows a difference in the right-hand side. Introducing surplus destruction increase the range of possible wage contracts in inefficient equilibria by exactly $\delta_F^{1-T}\psi$. But why does including surplus regeneration allow the union to demand even higher wage contracts? The union is not directly affected or threatened by the surplus regeneration of the firm—in contrast to surplus destroyed by the union, which clearly affects both players. Surplus regeneration only benefits the firm, whereas surplus destruction hurts the union as well as the firm since the surplus to be divided, Π_0, is destroyed. Intuitively, for a given period T of strikes, the associated surplus destruction now hurts the firm less than if surplus cannot be regenerated. Another way to put this, is that the union has to announce a longer strike the same wage contract, since the firm's pain threshold is now higher due to the payoff it receives from surplus regeneration.

This result may seem surprising at first, but if one thinks about it a little longer it makes sense. Even though the union is not directly affected by the surplus regeneration, we have introduced a game of complete and perfect information. That means that the union, when developing its strategy, considers the fact that the firm has the opportunity to regenerate surplus. It therefore knows that a specific strike hurts it less with surplus regeneration than without. So for the same strike, the union now may achieve an even higher wage contract, knowing that the firm will accept. Thus, surplus regeneration as introduced in our model may give the union incentives to strike for a longer time to reach a contract or to simply demand a higher wage contract. Note again that this is only the case for inefficient equilibria. For efficient equilibria, introducing surplus regeneration has no direct or indirect influence since, in that case, agreement is already reached in the first period. Since surplus regeneration cannot be seen as a threat towards the union, it does not change the union's equilibrium strategy for efficient equilibria.

5.3.2 The Busch, Shi & Wen model

On the other hand, if we put $\tau = 1$ we obtain the BSW model as a special case. To see this, we first have to fix the level of surplus destruction just as BSW did in their model. Thus, we put $\gamma_t = \gamma$ for all t and further we need to exclude surplus regeneration as part of the model. This is done by choosing a c such that $\hat{\alpha} = 0$, which therefore implies that $\alpha_t = 0$ for all t. Further, the transition equation (5.1) reduces to

$$\Pi_t = (\gamma + \alpha)^t \Pi_0 = \gamma^t \Pi_0$$

just as in the BSW model. Intuitively, by setting $\tau = 1$, there is no degree of reappearing surplus and therefore any destruction becomes purely permanent.

We wish to formulate a utility function that is equivalent to the original BSW utility. To do so, let T be the period of agreement and put

$$x_t = \begin{cases} 0, & \text{if } t < T \\ x_T, & \text{if } t \geq T \end{cases} \quad \text{and} \quad y_t = \begin{cases} 0, & \text{if } t < T \\ y_T = 1 - x_T, & \text{if } t \geq T \end{cases}.$$

The interpretation of x_t and y_t is that neither union nor firm gets paid any share before they reach agreement. Consider the firm's utility in (5.3). Obviously $\alpha\Pi_{t-1} = 0$ and, further, the first $T-1$ terms are 0 due to y_T. $\gamma_t = 1$ for all $t \geq T$ while $\gamma_t = \gamma$ otherwise. Hence the surplus left to split is $\gamma^T \Pi_0$. Analogously for the union and (5.3), reduces to:

$$\text{union: } \sum_{t=T}^{\infty} \delta_U^{t-1} \gamma^T \Pi_0 x_T \quad \text{and firm: } \sum_{t=T}^{\infty} \delta_F^{t-1} \gamma^T \Pi_0 (1 - x_T).$$

Thus, for given T the union will try to maximize x_T, while the firm will try to minimize x_T just as in the original BSW model. Hence, even though the utility function differs the maximization problem induces are equivalent, which is all that matters. Further, both players will prefer to reach agreement sooner rather than later due to the discount factor. That is they wish to reach agreement in as small a T as possible.

To get the introduction of surplus regeneration to be meaningful, we should be able to find at least one SPE involving $\alpha > 0$. We begin by analyzing this in the framework of the augmented BSW model. This should be thought of as the special case of our model just described, but where we allow $\alpha > 0$: that is, choose c such that $\hat{\alpha} > 0$. Alternatively, one can think in terms of the original BSW model and simply augment it by having the firm choose a feasible level of surplus regeneration after the union has chosen their surplus destruction. Note that (5.9) is the same requirement as (4.8) in Proposition 4.8, but we restate it here for convenience.

Lemma 5.1. *For fixed discount factors, let a* $\gamma \in [0,1]$ *exist such that*

$$\gamma b(\delta_U, \gamma\delta_F) \geq b(\delta_U, \delta_F). \tag{5.9}$$

Then an SPE exists in which the union threatens a level of surplus destruction of $1-\gamma$*, and the firm is ready to regenerate* $\alpha := \min\{a^*, 1-\gamma\}$ *of the surplus.*

This is important to understand and easily seen from the definition of the union's utility function. Any surplus regeneration does not affect the union's choices. Whether the firm chooses to regenerate surplus does not affect the utility of the union in the specific period. What it does affect is the bargaining strength of the union, as the maximum damage the union can inflict on the firm decreases. This means that the firm is vulnerable to surplus destruction by the union. Hence the union might not be able to demand as high a wage share as in the BSW model, or more generally, when $\tau < 1$, as if surplus regeneration were not introduced.

Proof. This proof is a modification of the proof of Proposition 4.8 and we only point out the differences. Consider the strategy: the union proposes $b(\delta_U, \gamma\delta_F)$, the firm rejects if and only if the union demands more, the union destroys $1-\gamma$ if rejected by the firm and the firm regenerates α. In an even period, let the firm propose $\delta_U b(\delta_U, \gamma\delta_F)$, the union rejects if and only if the firm offers less, let the union destroy $\gamma' = 1$ if it rejects the firm's offer and let the firm regenerate $\alpha = 0$. Again we use the Rubinstein equilibrium as the trigger strategy, but this time also if the firm deviates in its level of surplus regeneration. This implies that we only have to check for subgame perfection if a deviation occurs.

As already discussed, introducing surplus regeneration does not affect the union's best response. Hence for both odd and even periods the union does not have a profitable deviation as it is assumed (5.9). Consider the firm in both periods. The best response in terms of surplus regeneration will always by definition be exactly α. Thus, the firm is already playing its best response.

In conclusion, both the union and firm play their best responses, and the strategies considered must constitute a SPE. □

Besides justifying the introduction of surplus regeneration, the proof of Lemma 5.1 also shows that the union will never destroy surplus after rejecting the firm's proposal. Thus, this feature is inherited from the original BSW model.

We wish to investigate the lower bounds for the payoff of the union and firm. In the BSW model, payoff and share are synonymous, but this is not the case in our augmented model. The lower bound for the union continues to be the Rubinstein share, as it can always choose not to destroy any surplus, thereby eliminating the surplus regeneration option for the firm. The lower bounds for the firm can be analyzed by applying the Shaked & Sutton method [16].[8] Let L_1 and L_2 be the lowest payoff for the firm in Γ_U and Γ_F, respectively. In Γ_U, the highest payoff the union can expect is $1 - L_1$. Thus, the union will accept any offer in Γ_F that is at least $\delta_U(1-L_1)$. The firm is therefore guaranteed L_2 such that

$$L_2 \geq 1 - \delta_U(1 - L_1). \tag{5.10}$$

In Γ_U, if the union destroys $1-\gamma$, then the firm can choose to regeneration α in the following period at cost $c(\alpha)$, also paid in the following period. Thus, $\gamma + \alpha$ surplus

[8]Consult [6] "proof of proposition 4" on page 929.

is left, of which the firm can be sure to get L_2. We must discount all terms and obtain

$$L_1 \geq \delta_F \gamma L_2 + \delta_F \alpha L_2 - \delta_F c(\alpha). \tag{5.11}$$

Lastly, for the surplus destruction of the union to be credible, it must be

$$\gamma(1 - L_2) \geq \delta_U b(\delta_U, \delta_F). \tag{5.12}$$

From (5.10),(5.11) and (5.11), it follows that[9]

$$\frac{\delta_2 b(\delta_1, \delta_2)}{1 - L_1} \leq \frac{L_1 + \delta_2(c(\alpha) - \alpha(1 - \delta_1(1 - L_1)))}{1 - \delta_1(1 - L_1)}, \tag{5.13}$$

which is equivalent to

$$\frac{\delta_2 b(\delta_1, \delta_2)}{1 - L_1} - \frac{L_1 + \delta_2(c(\alpha) - \alpha(1 - \delta_1(1 - L_1)))}{1 - \delta_1(1 - L_1)} \leq 0, \tag{5.14}$$

If $\alpha = 0$, this reduces to the corresponding inequality in BSW, as $c(\alpha) = 0$. Note that, (5.14) defines a quadratic polynomial. We denote the roots by r_1 and r_2, respectively, and rewrite (5.14) as

$$a(L_1 - r_1)(L_1 - r_2) \leq 0,$$

where $a > 0$ is the coefficient of the quadratic term. It follows that $L_1 \geq \min\{r_1, r_2\}$, because, if not, $a(L_1 - r_1)(L_1 - r_2) > 0$, as $a > 0$. In the BSW model, the roots are simple and real numbers: $r_1 = 1 - \delta_F$ and $r_2 = 1 - b(\delta_U, \delta_F)$. Unfortunately, this is not the case in our model. The roots have a very lengthy algebraic expression and, even worse, some are complex. We therefore compared the lower bounds numerically where feasible; consult Appendix B. In Fig. 5.3 the points at which there is no difference in the lower bounds are plotted with red. These exactly correspond to a pair of discount factors for which destroying surplus is not feasible for the union. The interesting case is where the union does destroy surplus. As mentioned, for some pair of discount factors, the resulting equation has in complex roots. These pairs are plotted with purple. Frankly, the only positive thing to say about these is that they seem to be located in a connected area[10] rather than sporadically. Further, providing an economic interpretation of these complex roots is difficult. The two remaining areas depicts where the firm's lower bound is smaller (green) or greater (blue), respectively. These are connected as well, and it seems that a high discount factor for the union is correlated with a decrease in the firm's lower bound and vice versa.

5.3.3 The Rubinstein model

As we have seen, our model includes both the FG model and the BSW model as special cases. We know that the Rubinstein model is embedded in both of these, which means that the Rubinstein model is also imbedded in our generalization. But not only is it a special case, it is even a special case with two complete different sets of parameters, which indicates the very general approach our model permits.

[9] Consult Appendix A for calculations.
[10] Referring to the mathematical definition of connectedness.

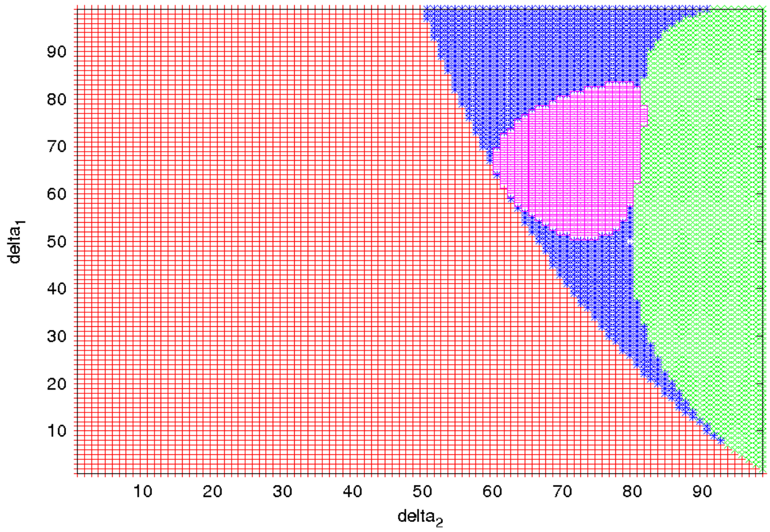

Figure 5.3: Plot of the difference between the BSW L_1 and the L_1 found in our model when surplus regeneration is added. δ_1 corresponds to δ_U and δ_2 to δ_F. They are given as fractions of 100. Red: no difference (compare to Fig. 4.1), purple: complex roots, green: smaller lower bound for firm and blue: greater lower bound for firm. Details on the plot can be found in Appendix B.

5.4 Analysis of the transition equation

In the previous section, we showed that the FG model and the BSW model are special cases of our model. Actually, they are located in the extremes, considering τ. τ is a major innovation of our model. The size of τ constitutes the degree of reappearing surplus: that is, a τ close to one means a low degree of reappearing surplus; surplus destruction affects subsequent surplus. A τ close to zero means a high degree of reappearing surplus. The following section shows the behavior of the surplus relative to τ.

5.4.1 Limit of convergence depending on τ

For convenience, define the sum $s_t = \gamma_t + \alpha_t$ as in the transition equation all that matters s_t. Whether a given level of s_t is feasible is a game theoretic question that is more easily analyzed independently. Generalizing the BSW approach, we fix γ_t and α_t and simply write $s = \gamma + \alpha$. Recall our transition equation (5.1). For fixed τ and s, we evaluate the limit of the surplus. Using iteration, we see that (5.1) reduces to

$$\Pi_t = s^{\sum_{i=0}^{t-1} \tau^i} \Pi_0 = s^{\frac{\tau^t - 1}{\tau - 1}} \Pi_0 \quad \text{for all } \tau \in [0, 1]. \tag{5.15}$$

Letting t move towards infinity, we define $\Pi_\infty = s^{\sum_{i=0}^{\infty} \tau^i} \Pi_0$. The sum in the exponent is a geometric series which can be rewritten as $\sum_{i=0}^{\infty} \tau^i = \frac{1}{1-\tau}$ for $\tau \neq 1$. Thus, the limit of Π_t can be expressed as

$$\Pi_\infty = s^{\frac{1}{1-\tau}} \Pi_0 \quad \text{for } \tau \neq 1. \tag{5.16}$$

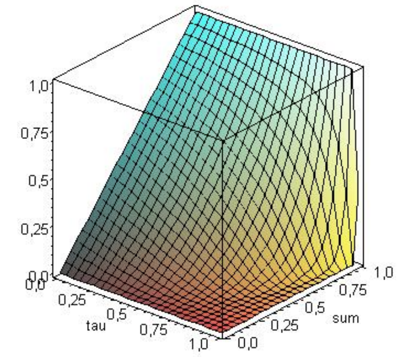

Figure 5.4: Plot of the surplus Π_∞ as a function of s and τ. Note that $\tau = 0$ amounts to being in the FG model and $\tau = 1$ amounts to being in the BSW model. Π_∞ is strictly greater than 0 for $\tau < 1$.

Fig. 5.4 shows a plot of the limit of the surplus as a function of τ and s. Some results should be explicitly specified. Remember that a low τ means a high reappearing surplus and a high τ a low reappearing surplus. By setting $\tau = 0$ in equation (5.16), we see that the limit in the FG model equals $\Pi_\infty = s\Pi_0$. Thus, the limit is a linear function of the sum s, which can be observed in Fig. 5.4. However, whenever $\tau > 0$, we see that the limit of the surplus becomes an exponential function of the sum s of degree $\frac{1}{1-\tau}$. Of interest is the case in which $\tau = 1$. This corresponds to the BSW model, which has no reappearing surplus except for $s = 1$. Thus, whenever $s < 1$, the limit of the surplus is zero.

5.4.2 Speed of convergence

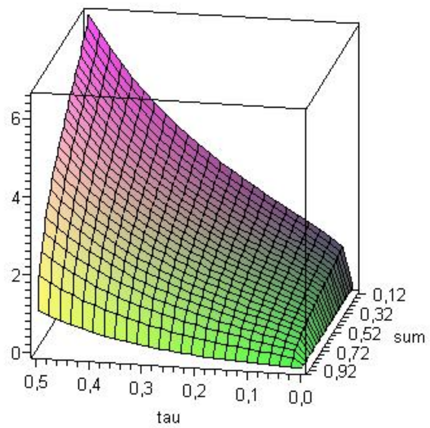

Figure 5.5: Plot of $S(r)$ with $r = 0.05$ for $\tau \in [0, 0.5]$

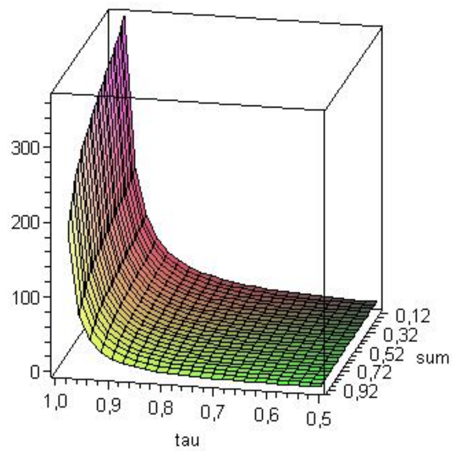

Figure 5.6: Plot of $S(r)$ with $r = 0.05$ for $\tau \in [0.5, 1]$.

Since we have analyzed the behavior of τ and τ's property of setting the level of reappearing surplus—that is, for every $0 \le \tau < 1$ the surplus in its limit is strictly greater than zero—it is interesting to examine the speed of convergence.

The speed of convergence is the number of periods it takes for fixed τ and s until the reappearing surplus is within a specific distance from its limit. For example, the smallest t such that $\Pi_t - \Pi_\infty \leq 0.05 \cdot \Pi_\infty$ gives the period in which we are 5% away. For a general $r > 0$

$$S(r) := \max\{0, \inf_t \{\Pi_t - \Pi_\infty \leq r \cdot \Pi_\infty\}\} \quad r > 0. \tag{5.17}$$

If $\Pi_t < (1+r)\Pi_\infty$, it is meaningless to ask for the smallest t such that we are within the limit. We therefore need to find the maximum between the infimum and 0 in (5.17). By inserting the values for Π_t and Π_∞ from (5.15) and (5.16), respectively, and solving for t, we arrive at

$$S(r) = t \geq \frac{1}{\log \tau} \log\left((1-\tau)\frac{\log(1+r)}{\log s}\right).$$

We call $S(r)$ the time of r-convergence. Fig. 5.5 and 5.6 are plots of $S(r)$ with $r = 0.05$. We see from Fig. 5.5, that the time of 0.05 convergence for small τ is really low. Intuitively, this makes sense, since our initial surplus is close to the limit surplus for small τ. However, for τ close to one, we observe asymptotic behavior. The z-axis of Fig. 5.6 shows a small fraction of the actual values of $S(r)$. We find that $\lim_{\tau \to 1} S(r) = \infty$ if $s < 1$. Thus, for $\tau = 1$ where $\Pi_\infty = 0$, $S(r)$ is not well defined.

CHAPTER 6

Perspective and conclusion

To generalize the FG and BSW approach, respectively, we introduced the transition equation. As FG and BSW use very different frameworks to describe their respective models, it is a nice and surprising discovery that both can be described using the same framework. This is mainly related to the more abstract approach of our model, as this allows us to choose x_t and y_t and also model to what degree the surplus in period t depends on the initial surplus. This naturally leads us to investigate the behavior of the surplus as a function of t. We have also examined the r-convergence—that is the rate of convergence—and were able to find an explicit functional expression that is easy to deal with in further analysis. Lastly, we have been able to show that any equilibria found in the FG and BSW model still exist in our model.

The second new feature we introduced was surplus regeneration by the firm. This has proved somewhat successful. We proved that, within the special cases given by FG and BSW, SPE exist that support surplus regeneration by the firm. In the BSW generalization, this was done by using the Rubinstein equilibrium as a trigger, and it is therefore essential that our model also supports a Rubinstein equilibrium for $\tau = 1$. We went one step further and tried to analyze the boundaries of the payoffs to the firm and the union. Unfortunately, this analysis could not be carried out algebraically, but analyzing the issue using Maple we found that the firm is better off in some cases and the union is better of in other cases. At first, it seems strange that the union should benefit from the firm being able to generate extra surplus, but we discuss this aspect in the following. In the FG generalization, we proceeded very similar to the case without surplus regeneration when finding SPE. The strategies used in efficient as well as inefficient equilibria were modified to include surplus regeneration by the firm. The trigger strategy is a feature used in the FG world as well. However, it does not have as much mathematical importance as in the BSW model. Similarly to the BSW approach, analyzing the boarders in which SPE are found in efficient equilibria is difficult. For inefficient equilibria, the boarders are easier to define: however, also here we found that the union benefits from the firm's surplus regeneration behavior.

Surplus regeneration was introduced as a semi-opting-out option, which only affected the firm. It was possible to regenerate a certain share of payoff α at a certain cost $c(\alpha)$ where the assumptions on c guaranteed nice behavior. We defined $\bar{\alpha}$ to be the amount of surplus regeneration resulting in the highest profit for the firm. By definition, the firm will play $\bar{\alpha}$ in any SPE.

We introduce surplus regeneration in this thesis in a naive manner. It only adds to the firm's payoff and does not inflict any kind of damage or negative utility on the union. The firm's choice of α is completely predictable after the union announces its level of surplus destruction. Thus, the union controls the firm's actions. Further, as the union receives a higher payoff when surplus regeneration is possible, it also means that the firm is indifferent to a higher level of the union's surplus destruction. The union is well aware of this and uses this to its advantage, thereby getting a higher upper bound on its payoff in many cases. This holds in every case in the FG model, whereas it is only true for certain pairs of discount factors in the augmented BSW model. In the analysis of this model, we sometimes find complex roots, which seems strange.

Intuitively, it would be nice to have a model with surplus regeneration that does not benefit the union in many or even most cases, as is true in our model. Although it was not clear to us a priori that this would be the result of the augmentation, backing up the results with intuition is not difficult considering the underlying dynamics as discussed above. We see two ways to remedy this weakness of the model. One uses a completely different approach from the other material presented throughout the thesis: surplus regeneration is private information to the firm. A model of asymmetric information would describe this, but we are not able to pursue this here.

The other approach is a more natural extension of this thesis. It would mean the surplus regeneration would negatively affect the union in some way. We believe that this could be justified by considering workers striking and, when they return to work, find out that the firm has permanently filled some positions with the freelance workers initially hired temporarily. Mathematically it would mean that the union would endure a cost in the period following the firm opting out, $c_U(\alpha)$. Although the union still knows exactly at what level the firm will regenerate surplus given the union's actions, the credible level of surplus destruction for the union will change. In the initial BSW model, the union sought to balance "a smaller share of a larger pie" versus "a larger share of a smaller pie". Introducing cost for the union, they would now have to balance "a smaller share of a larger pie" versus "a larger share of a smaller pie minus the cost of the firm opting out". We believe that there is a good chance this will give a more realistic description.

We want to turn back to the issue of the numerical analysis and complex roots when trying to find the bounds for the payoffs in the augmented BSW model. First, we had to fix a cost function and we chose the simple function $c(\alpha) = \alpha^2$. Using this set-up, we sometimes get complex roots, which is surprising. This makes any comparison of the bounds a mathematical impossibility, but worse, there seem to be no economic reason why these pair of discount factors should not give a feasible solution. Further, they seem to have a strange distribution. Despite this they can be characterized algebraically as complex roots appear if and only if the discriminant is strictly negative.

The area where no difference between the lower bounds is seen must be where destroying surplus is not credible for the union. In this case the firm cannot regenerate surplus and, as $c(0) = 0$, the lower bound of the augmented model must be identical to the original BSW lower bound. This condition is exactly the condition originally found by BSW when surplus destruction is feasible and is depicted in Fig. 4.1.

One area of further research could be the analysis of any other necessary constraint on the discount factors such that we can describe the four different kind of areas shown in the figure. We have a complete description of two of them, but the two interesting cases remain. All areas are well behaved, which gives hope to the fact that one can find a nice mathematical criterion to describe them. There seems to be a strong correlation between the union's discount factor and the firm's lower bound.

In this thesis and so far in this chapter we have only focused on $\tau \in \{0, 1\}$. It seems straightforward to see whether any of the above results could be generalized to $\tau \in (0, 1)$. Both the proof of the existence of a SPE and the analysis to find the lower bounds depends crucially on the existence of a Rubinstein equilibrium. It is interesting that this very simple model has served as a benchmark throughout all the models: those we analyzed as well as our attempts to generalize. Nevertheless, it therefore seems natural to start any further research by identifying Rubinstein equilibria for any given τ. These could then be used as part of a trigger strategy such that a equilibrium involving non-trivial surplus regeneration can be found, much like we did in the proof of Lemma 5.1 and throughout the equilibrium strategies of the FG model. This equilibrium can then be used to investigate the upper and lower bounds of the payoffs, which most likely will have to be done numerically, at least initially.

In our model we have not discussed inefficient equilibria. They are of great interest, but before these can be investigated we need a deeper understanding of the efficient equilibria as these are used as trigger strategies in any further analysis. We believe that the mathematical tools developed by BSW are suited for analyzing our generalization. However, much work still needs to be done before the inefficient equilibria can be considered, not to mention the analysis of real time delay or a folk theorem. Nevertheless, we believe that the foundations of our model is solid and has potential. Our findings make a modest beginning, but there are still questions of interest left to investigate. However, these lie beyond the scope of this thesis.

APPENDIX A

Mathematical tools and calculations

This appendix presents theorem and calculations needed throughout the thesis, but more suitable to state in an appendix.

Kakutani's fixed point theorem

Denote the power set of X by $\mathcal{P}(X)$. A function that maps from X to $\mathcal{P}(X) \setminus \{\emptyset\}$ is called a set-valued function and is written $f : X \to X$. It is a way to remedy the technical definition of a function, which only allows $x \in X$ to be mapped to one element. By mapping to sets instead, f can be considered as a multivalued function.
Theorem A.1 (Kakutani's fixed point theorem)**.** *Let X be a compact convex subset of $\mathbb{R}^n$ and let $f : X \to X$ be a set-valued function for which*

i) for all $x \in X$ the set $f(x)$ is nonempty and convex; and

ii) the graph of f is closed.

Then f has a fixed point; that is there exists an x^* such that $x^* \in f(x^*)$.

To understand what the graph of f being closed means, it is useful to remember the more technical definition of a set-valued function: the graph is considered closed if $\{(x, y) : x \in X, y \in f(x)\}$ is a closed subset of $X \times X$.

Proof. Consult [17] for a proof. □

Calculation of (5.13)

The key to establishing the result inequality in (5.13) is to find an upper and lower bound for $\gamma\delta_F$. From (5.10) it follows that

$$\frac{L_1 - \delta_F(\alpha L_2 - c(\alpha))}{L_2} \geq \gamma\delta_F,$$

and by applying (5.10) both in the denominator and numerator, we obtain

$$\gamma\delta_F \leq L_1 + \delta_2(c(\alpha) - \alpha(1 - \delta_1(1 - L_1)))1 - \delta_1(1 - L_1).$$

This gives the upper bound for $\gamma\delta_F$. What remains is the lower bound, and from (5.12), we find

$$\frac{\delta_F \delta_U b(\delta_U, \delta_F)}{1 - L_2} \leq \gamma\delta_F,$$

and (5.10) gives us $1 - L_2 \leq \delta_U(1 - L_1)$. Inserting in the above gives the lower bound

$$\frac{\delta_F b(\delta_U, \delta_F)}{1 - L_1} \leq \gamma\delta_F.$$

This concludes the calculations needed to establish (5.13).

APPENDIX B

Comparison of lower bounds in our model and BSW model

This appendix compares the lower bounds found analytically in the BSW model with the lower bounds found numerically in our model. To plot the calculations it is easier to have data in four different data files: `case1.data`, `case2.data`, `case3.data` and `case4.data`. Calculations are done in Maple using the following code.

```
restart;
#INITIAL SETUP. n IS THE NUMBER OF POINTS ANALYZED ON EACH AXIS
c:= alpha-> alpha**2: n:=100:

gamma_star := proc (delta1,delta2)
  min(1,(1-delta1)/(delta2*(1-delta1*delta2)));
end proc:

L_1_BSW := proc (delta1,delta2)
  min(1-delta2,delta2*(1-delta1)/(1-delta1*delta2));
end proc:

L_1_SR := proc (d1,d2,alpha)
  solve(d2*(1-d2)/((1-L)*(1-d1*d2))-(L+d2*c(alpha))/(1-d1*(1-L))
    +d2*alpha=0,L)
end proc:

#GENERATING THE DATA AND WRITING IT TO 4 SEPARATE FILES
fopen("case1.data",WRITE,TEXT):fopen("case2.data",WRITE,TEXT):
  fopen("case3.data",WRITE,TEXT):fopen("case4.data",WRITE,TEXT):
for j from 1 to n-1 do
  for k from 1 to n-1 do
    B:=0:
    #DETERMINING THE LEVEL OF SURPLUS RECREATION
    if (gamma_star(j/n,k/n)<1) then a:=min(1-gamma_star(j/n,k/n),
      0.5) else a:=0 end if:
    if (gamma_star(j/n,k/n)=1) then fprintf("case1.data","%d %d
```

```
      %f\n",j,k,0
      ) end if:
    #SOLVING THE QUADRATIC EQUATION
    A:=L_1_SR(j/n,k/n,a):
    #COMPARING BSW AND OUR MODEL IF ROOTS ARE REAL.
    if (Im(A[1])=0) then B:=L_1_BSW(j/n,k/n)-min(A) else fprintf(
      "case4.data","%d %d %s\n",j,k,"Imaginary Roots") end if;
    if (B>0) then fprintf("case2.data","%d %d %f\n",j,k,B) end if;
    if (B<0) then fprintf("case3.data","%d %d %f\n",j,k,B) end if;
  end do;
end do;
fclose("case1.data"):fclose("case2.data"):fclose("case3.data"):
  fclose("case4.data")
```

First procedures for given $(\delta_U, \delta_F) = (\delta_1, \delta_2)$ are defined to find the optimal level of surplus destruction for the union, `gamma_star`, the BSW lower limit for L_1, `L_1_BSW`, and the lower limit L_1 in our model with surplus regeneration, `L_1_SR`. The second part of the code is a double for-loop such that δ_1 and δ_2 runs through the set $\{1/n, \ldots, (n-1)/n\}$. `B` is a local dummy variable. $\alpha^* = 1/2$ as $c(\alpha) = \alpha^2$ and therefore if the union actually does perform surplus destruction the firm will recreate $\alpha = \min\{1-\gamma^*, 1/2)$ and $\alpha = 0$ otherwise. The four data-files make up a partition of the discount factors. If a pair of discount factors does not support surplus destruction they are written to `case1.data`. `case2.data` and `case3.data` record `B>0` and `B<0`, respectively. Any pair giving complex roots are written to `case4.data`.[1]

The plot found in Fig. 5.3 is generated with Gnuplot. For more information visit: `http://gnuplot.info/`.

Maple-file and data-files can be found online:
`http://baltzersen.info/mathecon_bachelor.php`.

[1]Note that either both roots are real or both roots are imaginary as the coefficients of the quadratic polynomial are real.

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